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Active self-sensing scheme development for structural health monitoring

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Abstract

Smart materials such as lead zirconate titanate (PZT) have been widely used for generating and measuring guided waves in solid media. The guided waves are then used to detect local defects for structural health monitoring (SHM) applications. In this study, a self-sensing system, composed of self-sensing algorithms and a self-sensing circuit equivalent to a charge amplifier, is developed so that a single PZT wafer can be used for simultaneous actuation and sensing. First, a PZT wafer is modeled as a single capacitor and a voltage source, and a so-called scaling factor, defined as the ratio of the PZT capacitance to the capacitance of the feedback capacitor in the self-sensing circuit, is estimated by applying known waveforms to the PZT wafer. Then, the mechanical response of the PZT wafer coupled with the host structure’s response is extracted from the measured PZT output voltage when an arbitrary excitation is applied to the same PZT wafer. While existing self-sensing techniques focus on vibration controls, the proposed self-sensing scheme attempts to improve the accuracy of extracted sensing signals in the time domain. The simplicity, adaptability and autonomous nature of the proposed self-sensing scheme make it attractive for continuous monitoring of structures in the field. The effectiveness of the proposed self-sensing scheme is investigated through numerical simulations and experiments on a cantilever beam.

1. Introduction

Within our society, there are increasing demands to adopt structural health monitoring (SHM) and non-destructive testing (NDT) technologies for monitoring and maintenance of aerospace, civil infrastructure and mechanical systems. Recently guided waves have been widely used for SHM and NDT applications [1–4]. All elastic waves including body and guided waves are governed by the same set of partial differential equations. The primary difference is that, while body waves are not constrained by any boundaries, guided waves need to satisfy the boundary conditions imposed by the physical systems as well as the governing equations [5]. Because of this ‘guided’ nature, guided waves can propagate a relatively long distance with little attenuation, thus providing a sensing range which is in between those of conventional NDT techniques and global SHM techniques. Wafer-type piezoelectric materials such as lead zirconate titanate (PZT) are widely used for exciting and measuring guided waves in SHM and NDT applications. One of the important characteristics of the PZT materials is that they can be used for simultaneous sensing and actuation. This feature of the PZT materials is called self-sensing here, and the advantages of a self-sensing scheme include: (1) the pulse-echo time reversal method can be achieved using only a single PZT wafer [6], (2) sensor diagnosis schemes can be implemented based on self-sensing to monitor sensors’ performance and integrity [7, 8], (3) a single PZT wafer can be used to suppress undesired vibrations [10, 15].

A self-sensing concept using a PZT wafer was investigated by several researchers including Anderson et al [9] and Dosch et al [10]. They used a bridge circuit to extract the mechanical
response of a structure from the PZT output voltage by manually matching the capacitance of the reference capacitor with that of the PZT wafer. However, because the ambient variations of the structure, such as temperature and humidity, can change the capacitance value of the PZT wafer, Cole and Clark proposed an adaptive compensation method to estimate the capacitance of the piezoelectric wafer using LMS (least mean squares) and RLS (recursive least squares) adaptive filters [11]. Then, Vipperman and Clark implemented the aforementioned adaptive filters on a DSP (digital signal processing) chip, effectively measured the PZT capacitance, and successfully applied the adaptive filtering to vibration suppression [12]. Okugawa and Sasaki identified the transfer function of a self-sensing system using a subspace state space identification method (software tuning) instead of a bridge circuit (hardware tuning) [13]. Law et al developed another adaptive mechanism by combining a bridge circuit with a positive position feedback controller for real-time vibration suppression [14]. Note that the majority of this previous work on self-sensing has focused on real-time vibration control, emphasizing the frequency content of the mechanical response.

On the other hand, the ultimate goal of this study is to apply self-sensing to guided wave based damage detection and PZT transducer diagnosis. In particular, this study focuses on estimation of the mechanical response in the time domain for system and transducer diagnosis applications. The biggest challenge of this self-sensing comes from the fact that the magnitude of the mechanical response is generally several orders of magnitude smaller than that of the input signal. The proposed self-sensing takes full advantage of the facts that any user-defined input signals can be applied to a structure using a PZT transducer and the input waveform is known a priori. Additional advantages of the proposed approach are its simplicity and adaptability. Only additional hardware required is a self-sensing circuit equivalent to a charge amplifier, and the self-sensing parameters can be calibrated instantaneously at the presence of changing operational and environmental conditions of the system.

This paper is organized as follows. First, the proposed self-sensing scheme, which is composed of a self-sensing circuit and self-sensing estimation techniques, is theoretically developed. Then, numerical simulations and experimental studies are performed to demonstrate the effectiveness of the proposed self-sensing scheme. Finally, this paper concludes with a brief summary and discussions for future work.

2. Theoretical development

This section develops the theoretical framework of the proposed self-sensing scheme. Figure 1 shows the schematic of the proposed self-sensing circuit and a cantilever beam used in this study. A single PZT wafer is mounted on one surface of the beam near the fixed end. The free surface of the PZT wafer is connected to a voltage source \( v_i \) from an arbitrary signal generator, and the other surface is tied to a self-sensing circuit equivalent to a charge amplifier. Then, the output from the self-sensing circuit \( v_o \) is connected to a signal digitizer. Finally, the proposed self-sensing algorithms use the known input signal \( v_i \) and the measured output signal \( v_o \) to extract the mechanical response of the structure \( v_p \).

The proposed self-sensing scheme consists of two main steps: (1) calibration of the self-sensing circuit by applying a probing waveform and (2) extraction of a mechanical response corresponding to an arbitrary input waveform. The first step (step I) is to measure a so-called scaling factor, which is defined as the ratio of the PZT capacitance to that of the feedback capacitor in the self-sensing circuit. The proposed scaling factor is estimated by three different algorithms: root mean square (RMS), LMS and orthogonality algorithms. These three algorithms will be described in detail in section 2.3. In the second step (step II), the mechanical response corresponding to an arbitrary input signal is estimated using the previously calibrated self-sensing circuit.

First, the piezoelectricity model of the PZT wafer and its interaction with a cantilever beam is formulated to understand the relation of the PZT input and output voltages with the vibration characteristics of the beam. Second, the self-sensing circuit is analyzed to show how the output voltage of the self-sensing circuit is related to the input and mechanical response voltages. Then, three different algorithms for estimating the mechanical response are compared, and error analysis is performed to identify the best estimation algorithm.

2.1. PZT wafer model with beam structure

To understand how the input voltage applied to the PZT wafer introduces the mechanical response of the beam and subsequently the electric charge in the PZT wafer, a piezoelectricity model of the PZT wafer coupled with the differential equation of the beam’s vibration is derived here. The stress, strain, electric field, and electric displacement within a piezoelectric material can be described by a pair of electromechanical equations [16]:

\[
\begin{align*}
S_1 &= s_{11}^E T_1 + d_{31} E_3 \\
D_1 &= d_{31} T_1 + s_{33}^E E_3
\end{align*}
\]

where \( S_1 \) is the \( x \)-directional strain, \( s_{11}^E \) is the \( x \)-directional compliance of the PZT wafer, \( T_1 \) is the \( x \)-directional stress, \( d_{31} \) is the induced strain coefficient, \( E_3 \) is the \( z \)-directional electric field, \( D_3 \) is the \( z \)-directional electric displacement, and \( s_{33}^E \) is the \( z \)-directional dielectric permittivity. Note that the coordinate system is defined in figure 1.

When the PZT wafer is attached to the structure, the electric field applied in the \( z \)-direction of the PZT wafer...
generates the strain in the $x$-direction and consequently the vibration of the beam. The dynamic behavior of the cantilever beam subject to the PZT excitation can be described using the following Euler–Bernoulli beam equation \[17\]:
\[
\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = \frac{\partial^2 M_{\text{PZT}}(x,t)}{\partial x^2} \tag{2}
\]
where $w(x,t)$ is the $z$-directional displacement, $\rho$ is the material density, $A$ is the cross-sectional area, $E$ is Young’s modulus, $I$ is the moment of inertia, and $M_{\text{PZT}}$ is the moment produced by the PZT wafer. The moment $M_{\text{PZT}}$ generated by the PZT wafer can be described as
\[
M_{\text{PZT}}(x,t) = \int_{z_1}^{z_2} T_i b z \, dz = \int_{z_1}^{z_2} Y_d d_3 b \frac{v_i(t)}{I_0} b z \, dz = \frac{Y_d d_3 b}{I_0} \left( \frac{z_2^2 - z_1^2}{2} \right) v_i(t) = K_a v_i(t) \left[ H(x - x_1) - H(x - x_2) \right] \quad \text{and} \quad K_a = \frac{Y_d d_3 b}{I_0} \left( \frac{z_2^2 - z_1^2}{2} \right) \tag{3}
\]
where $Y_d$ is Young’s modulus of the PZT wafer, $I_0$ and $b$ are the thickness and the width of the PZT wafer, and $H(x)$ is a Heaviside step function, respectively. Note that $z_1$ and $z_2$ are defined in figure 1. Using separation of variables, $w(x,t)$ in equation (2) can be separated into spatial variable $X_n(x)$ and time variable $Q_n(t)$ as
\[
w(x,t) = \sum_{n=1}^{\infty} X_n(x)Q_n(t) \quad \text{where} \quad \int_{0}^{l} X_n^2(x) \, dx = 1 \tag{4}
\]
where $X_n(x)$ is the normalized $n$th mode eigenfunction obtained by solving the homogeneous equation of equation (2) with respect to $x$, and $l$ is the length of the beam. Then, the response of the $n$th modal basis becomes
\[
\ddot{Q}_n(t) + 2\xi_n \omega_n \dot{Q}_n(t) + \omega_n^2 Q_n(t) = \frac{K_a W_n}{\rho A} v_i(t) \tag{5}
\]
where $\dot{Q}_n(t)$ and $\ddot{Q}_n(t)$ are the first and second derivative of $Q_n(t)$ with respect to time, respectively. $\xi_n$ is the $n$th mode damping ratio, $\omega_n$ is the $n$th resonance angular frequency, $W_n = X_n'(x_2) - X_n'(x_1)$ and $X_n'(x)$ is the first derivative of the normalized $n$th mode eigenfunction with respect to $x$. The voltage generated from the mechanical response can be found as
\[
v_p(t) = \frac{q}{C_p} = \frac{d_331 Y_d z_c}{C_p} \left[ w'(x_2,t) - w'(x_1,t) \right] = K_s \sum_{n=1}^{\infty} Q_n(t) \cdot W_n \tag{6}
\]
where $C_p$ is the capacitance of the PZT wafer, $z_c$ is the distance from the neutral axis of the beam, and $K_s = Y_d d_3 b z_c / C_p$. Equation (6) shows that the PZT strain produces the bending of the beam, and the beam’s deflection in turn generates the mechanical response voltage at the PZT wafer.

2.2. Circuit model with PZT wafer

Next, the output voltage of the self-sensing circuit is related to the input and mechanical response voltages of the PZT wafer. As shown in figure 1, the proposed self-sensing circuit is equivalent to a charge amplifier. From the circuit diagram shown in figure 1, the output of the circuit is related to the input and mechanical voltages of the PZT wafer as follows:
\[
\dot{v}_i(t) = C_p \left[ \dot{v}_o(t) + \dot{v}_p(t) \right] = -C_i \dot{v}_o(t) - \frac{v_o(t)}{R} \tag{7}
\]
\[
\dot{v}_o(t) + \frac{v_o(t)}{RC_i} = -\frac{C_p}{C_i} \left[ \dot{v}_i(t) + \dot{v}_p(t) \right] \tag{8}
\]
where $R$ and $C_i$ are the feedback resistor and the feedback capacitor of the self-sensing circuit, respectively. It is shown that the output from the self-sensing circuit is related to the input and mechanical response of the PZT wafer as well as the PZT and reference capacitor capacitance values. When a sinusoidal input $v_i(t) = V \sin(\omega t)$ is applied to the PZT wafer, the response of the $n$th modal basis can be obtained from equation (5) as follows:
\[
Q_n(t) = \frac{K_a W_n V (\omega_n^2 - \omega^2)}{\rho A \left( (\omega_n^2 - \omega^2)^2 + (2\xi_n\omega_n\omega)^2 \right)} \sin \omega t - \frac{2K_a W_n V \xi_n\omega_n}{\rho A \left( (\omega_n^2 - \omega^2)^2 + (2\xi_n\omega_n\omega)^2 \right)} \cos \omega t. \tag{9}
\]
As the driving frequency $\omega$ increases, the coefficients of $Q_n(t)$ converge to zero and the term $\dot{v}_o(t)$ in equation (6) vanishes. The physical interpretation of this is that there is little mechanical response as the driving frequencies increase. Then, the steady-state solution of equation (8) becomes
\[
v_o(t) = \frac{C_p}{C_i} \frac{\omega^2 R^2 C_i^2 + 1}{\omega^2 R^2 C_i^2 + 1} V \sin \omega t - \frac{C_p}{C_i} \frac{\omega R C_i}{\omega^2 R^2 C_i^2 + 1} V \cos \omega t. \tag{10}
\]
As the driving frequency $\omega$ increases, the relationship between the output and input voltages can be simplified:
\[
v_o(t) \simeq -\frac{C_p}{C_i} V \sin \omega t = -\frac{C_p}{C_i} v_i(t) \tag{11}
\]
where the scaling factor of the proposed self-sensing circuit is defined as the ratio of $-C_p$ to $C_i$:
\[
SF = -\frac{C_p}{C_i} \simeq \frac{v_o(t)}{v_i(t)}. \tag{12}
\]
Equation (12) indicates that the scaling factor can be approximated by computing the amplitude ratio of the output voltage to that of the input voltage when the driving frequency is high enough. This approximation is used in the following subsection to estimate the scaling factor from the input and output voltages.

2.3. Scaling factor estimation algorithms

In this subsection, three different algorithms for estimating the scaling factor are derived and their susceptibility to Gaussian white noise in the input and output voltages is investigated.
2.3.1. RMS algorithm. First, the scaling factor is estimated by computing the ratio of the RMS value of the output signal to that of the input signal as follows:

\[
SF_{\text{RMS}} = - \frac{\text{RMS}[\tilde{v}_o[k]]}{\text{RMS}[\tilde{v}_i[k]]}
\]

where \( \tilde{v}_i[k] \) and \( \tilde{v}_o[k] \) denote noise-contaminated versions of the input and output signals and are defined as \( \tilde{v}_i[k] = v_i[k] + e_i[k] \) and \( \tilde{v}_o[k] = v_o[k] + e_o[k] \), respectively. \( e_i[k] \) and \( e_o[k] \) are output and input Gaussian white noises. Furthermore, \( \tilde{v}_i[k] \) is a discrete version of the continuous signal \( v_i(t) \) and is defined as \( \tilde{v}_i[k] = v_i(k \times \Delta t) \). Here, \( \Delta t \) is the sampling time. \( v_i[k] \) is defined in a similar fashion. The RMS of the input signal can be computed as

\[
\text{RMS}[\tilde{v}_i[k]] = \sqrt{\frac{1}{m} \sum_{k=0}^{m-1} \tilde{v}_i[k]^2} = \sqrt{\frac{1}{m} \sum_{k=0}^{m-1} (v_i[k] + e_i[k])^2}
\]

\[
\text{RMS}[\tilde{v}_i[k]] = \sqrt{\frac{1}{m} \sum_{k=0}^{m-1} (v_i[k] + e_i[k])^2} = \sqrt{\frac{1}{m} \sum_{k=0}^{m-1} (v_i[k] + e_i[k])^2}
\]

\[
\text{RMS}[\tilde{v}_i[k]] = \sqrt{\frac{1}{m} \sum_{k=0}^{m-1} (v_i[k] + e_i[k])^2}
\]

From equations (13) to (15), it can be shown how the scaling factor estimation is influenced by the input and output noises as well as the mechanical response.

2.3.2. LMS algorithm. From [11], an LMS algorithm is applied to estimate the scaling factor by calculating the ratio of the cross correlation between the output and input signals to the autocorrelation of the input signal as follows:

\[
SF_{\text{LMS}} = \frac{E \{ \tilde{v}_o[k] \cdot \tilde{v}_i[k] \}}{E \{ \tilde{v}_i[k] \cdot \tilde{v}_i[k] \}}
\]

The autocorrelation of the input signal is defined as

\[
E \{ \tilde{v}_i[k] \cdot \tilde{v}_i[k] \} = E \{ (v_i[k] + e_i[k]) \cdot (v_i[k] + e_i[k]) \} = E \{ v_i[k]^2 \} + 2E \{ v_i[k] \cdot e_i[k] \} + E \{ e_i[k]^2 \}
\]

\[
E \{ v_i[k] \} = E \{ v_i[k] + e_i[k] \} = E \{ v_i[k] \} + E \{ e_i[k] \}
\]

where \( E \{ v_i[k] \} \) is the expectation operator defined as \( E \{ v_i[k] \} = \sum_{k=0}^{m-1} v_i[k]/m \). The cross correlation between the input and output signals is defined similarly:

\[
E \{ \tilde{v}_o[k] \cdot \tilde{v}_i[k] \} = E \{ (v_o[k] + e_o[k]) \cdot (v_i[k] + e_i[k]) \} = E \{ v_o[k] \cdot v_i[k] \} + E \{ v_o[k] \cdot e_i[k] \} + E \{ e_o[k] \cdot v_i[k] \} + E \{ e_o[k] \cdot e_i[k] \}
\]

\[
E \{ v_o[k] \cdot v_i[k] \} = E \{ v_o[k] \} \cdot E \{ v_i[k] \}
\]

\[
E \{ v_o[k] \cdot e_i[k] \} = E \{ v_o[k] \} \cdot E \{ e_i[k] \}
\]

\[
E \{ e_o[k] \cdot v_i[k] \} = E \{ e_o[k] \} \cdot E \{ v_i[k] \}
\]

\[
E \{ e_o[k] \cdot e_i[k] \} = E \{ e_o[k] \} \cdot E \{ e_i[k] \}
\]

Because the input and output noises are assumed to be Gaussian white noises, they are independent of the output and input voltages as well as of each other. Furthermore, by choosing a high-frequency input signal, the mechanical response can be minimized and the error due to the cross correlation term \( E \{ v_i[k]v_o[k] \} \) becomes negligible. Therefore, when the scaling factor is estimated using the LMS algorithm, only the error due to the Gaussian input white noise remains in the denominator of equation (16).

2.3.3. Orthogonality algorithm. In the orthogonality method, the numerator and denominator in equation (12) are first multiplied by a sinusoidal wave at the input frequency. Then, the numerator and denominator are summed over the entire length of the signal as follows:

\[
SF_{\text{ORT}} = \frac{\sum_{k=0}^{m-1} \tilde{v}_o[k] \cdot \sin(\omega_0 k \Delta t)}{\sum_{k=0}^{m-1} \tilde{v}_i[k] \cdot \sin(\omega_0 k \Delta t)}
\]

The numerator in equation (19) becomes

\[
\sum_{k=0}^{m-1} \tilde{v}_o[k] \cdot \sin(\omega_0 k \Delta t) = \sum_{k=0}^{m-1} (v_o[k] + e_o[k]) \cdot \sin(\omega_0 k \Delta t)
\]

\[
= \sum_{k=0}^{m-1} v_o[k] \cdot \sin(\omega_0 k \Delta t).
\]

The numerator in equation (19) becomes

\[
\sum_{k=0}^{m-1} \tilde{v}_o[k] \cdot \sin(\omega_0 k \Delta t)
\]

\[
= \sum_{k=0}^{m-1} \left\{ \frac{C_p}{C_t} (v_o[k] + v_p[k]) + e_o[k] \right\} \cdot \sin(\omega_0 k \Delta t)
\]

\[
= \sum_{k=0}^{m-1} \left\{ \frac{C_p}{C_t} (v_o[k] + v_p[k]) \right\} \cdot \sin(\omega_0 k \Delta t).
\]

Since the orthogonality algorithm uses the ideal sinusoidal signal that does not have a noise term, the orthogonality method is expected to be less susceptible to input and output noises. Similar to the LMS algorithm, the cross correlation term \( E \{ v_p[k] \cdot \sin(\omega_0 k \Delta t) \} \) in equation (21) remains insignificant as long as the amplitude of \( v_p[k] \) at the driving frequency is negligible. Note that the response component of \( v_p[k] \) outside the driving frequency is cancelled out through the orthogonality of trigonometric signals. Table 1 shows the comparison between the three algorithms derived in the previous subsection and suggests that the orthogonality algorithm has the best performance.
2.4. Mechanical response extraction

In the previous section, the scaling factor is estimated using three different estimation algorithms, and their susceptibility to random input and output noises is investigated. Here, a mechanical response is extracted using the previously estimated scaling factor and by applying an arbitrary input waveform. It is also analyzed how the error in the scaling factor estimate affects the estimation of the mechanical response.

First, the relative error in the scaling factor estimate is defined as

\[ e = \frac{|\hat{SF} - SF|}{SF} \]  

(22)

where \( \hat{SF} \) is the estimated scaling factor and \( SF \) is the exact scaling factor. The output voltage from the self-sensing circuit can be related to the PZT input voltage and the mechanical response based on equation (8). As a typical value of the feedback resistor in equation (8) is large, the second term in the left-hand side of equation (8) can be neglected. Then, by taking the definite integral of the simplified version of equation (8), the following equation is obtained under the assumption that the exact scaling factor is known:

\[ v_o[k] = SF \cdot (v_i[k] + v_p[k]) \Rightarrow v_p[k] = \frac{1}{SF} v_o[k] - v_i[k]. \]  

(23)

However, because the exact scaling factor is unknown in real applications, the mechanical response can be estimated by dividing the measured output voltage by the estimated scaling factor and subtracting the input signal from it:

\[ \hat{v}_p[k] = \frac{1}{\hat{SF}} (v_o[k] - \hat{SF} \cdot v_i[k]) \]
\[ = \frac{1}{\hat{SF}} (SF \cdot (v_i[k] + v_p[k]) - \hat{SF} \cdot v_i[k]) \]
\[ = \frac{1}{1 + e} (v_i[k] + v_p[k]) - v_i[k] \]
\[ = \frac{1}{1 + e} v_p[k] - \frac{e}{1 + e} v_i[k]. \]  

(24)

Next, the error between the actual mechanical response and the measured mechanical response is defined as the ratio of the RMS difference between them to the RMS of the actual mechanical response:

\[ \text{RMS}(\hat{v}_p[k] - v_p[k]) \]
\[ \text{RMS}(v_p[k]) \]

\[ \frac{\sum_{k=0}^{m-1} (\hat{v}_p[k] - v_p[k])^2}{\sum_{k=0}^{m-1} v_p[k]^2}. \]  

Equation (25) shows how the error between the actual mechanical response and the measured mechanical response is related to the error of the scaling factor estimate, the actual mechanical response, and the input signal. It is shown that the error in the measured mechanical response can be minimized by reducing the scaling factor error and by increasing the amplitude of the mechanical response with respect to the input signal.

3. Numerical simulation results

This section reports the results of numerical simulations that were performed to compare the performances of the three self-sensing algorithms and investigate the effect of the scaling factor accuracy on the extracted mechanical response. The beam and circuit model shown in figure 1 were numerically modeled using Simulink®. The dimensions of the aluminum beam and the PZT wafer are 510 mm × 19 mm × 3 mm and 72.5 mm × 19 mm × 0.508 mm, respectively. The detailed parameters for the simulation are shown in table 2. The material properties of the PZT wafer are identical to those of PZT-5A (Navy type II) [18]. For the simulation, only the first four bending modes of the beam were considered in equation (6). The first four resonance bending modes of the beam are identified to be 9.419 Hz, 59.028 Hz, 165.28 Hz, 323.88 Hz, respectively, and the corresponding normalized mode shapes are shown in figure 2. Note that each mode shape is normalized such that its maximum amplitude equals to 1. To simulate measurement uncertainty, Gaussian white noise is added to the input and output signals. The standard deviation (STD) of the Gaussian white noise was determined by measuring the noise level of the data acquisition system used in the later experiments as shown in table 2. Then, the scaling factor was estimated using a high-frequency sine input signal in step I, and the corresponding mechanical response was extracted in the time domain in step II.

In step I, the scaling factors were estimated using the three different estimation algorithms, and the relative errors with respect to the exact scaling factor are shown in table 3. Among the three algorithms, the orthogonality algorithm produced the minimum error in the scaling factor estimation. These
output signal from the self-sensing circuit was measured. For step II, a chirp signal starting from 0 Hz and ending at 400 Hz was applied as the input signal, and the corresponding frequency content of the chirp signal is −4 V 0–400 Hz chirp. Note that the frequency domain response shown in figure 3(f). It should be noted that the scaling factor error only alters the anti-resonance frequencies of the system (system zeros) but do not affect the resonance frequencies (system poles). This can be explained by examining equation (24). By taking a discrete Fourier transform of equation (24), it can be shown that the error in the scaling factor affects the numerator of the transfer function and subsequently changes the positions of the system zeros (anti-resonance frequencies). On the other hand, the scaling factor does not alter the values of the system poles (resonance frequencies) although their amplitudes are scaled by 1/(1 + e). Therefore, the resonance frequencies of the mechanical response can be reliably estimated even with the presence of the error in the scaling factor.

4. Experimental test results

This section reports on the experimental tests performed to verify the theoretical development and to substantiate the findings from the numerical simulation results. In particular, the performances of the three self-sensing algorithms are compared, and the effect of the scaling factor error on the extracted mechanical response is investigated. For step I, a high-frequency sine wave is used as the input signal and the scaling factor is estimated by the three different algorithms. For step II, the mechanical response in the time domain and the corresponding frequency spectrum are estimated. In order to show the effect of the scaling factor estimate error on the accuracy of the extracted mechanical response, five different levels of the scaling factor estimate error are examined similar to the numerical simulation. Then, additional experiments are performed to examine how the self-sensing circuit dynamics and the estimation error in the scaling factor affect the mechanical response estimation.

4.1. Self-sensing scheme

4.1.1. Experimental set-up. Waveforms applied to the PZT wafer were generated by an arbitrary waveform generator that had 16-bit resolution and 100 Ms s⁻¹ sampling rate.
Figure 3. Estimated time and frequency domain responses of the test structure when a chirp input signal (0–400 Hz) is applied with five different error cases in the scaling factor estimation from −2.5% to 2.5%. (a) A chirp input signal in the time domain. (b) The frequency content of the input signal. (c) The simulated output voltage in the time domain. (d) The frequency spectrum of the output voltage. (e) The extracted mechanical time response. (f) The frequency spectrum of the extracted mechanical response.

Then, the self-sensing circuit was built on a bread-board with commercial resistors and capacitors. Finally, the input and output signals were measured by a signal digitizer that supported 14-bit resolution and 100 Ms s$^{-1}$ sampling rate. The operation of the signal generator and the digitizer was controlled by commercial software, LabVIEW. Figure 4 shows the actual experimental set-up except for the data acquisition system. To verify the results of the scaling factor estimate, the capacitance values of the PZT wafer and the reference capacitor were initially measured by a commercial LCR meter, which had 0.05% accuracy. From the LCR meter, the capacitance values of the PZT wafer and the reference capacitor in the self-sensing circuit were estimated to be 32.945 and 39.864 nF, respectively. Without using any additional low-pass filter or power amplifier, the same excitation signal was applied ten times, and the corresponding signals were averaged to improve the signal-to-noise ratio. A time interval of about 20 s was taken between two adjacent input excitations to minimize vibration interference among subsequent excitations. The rest of the experimental parameters were kept same as those of the numerical simulation.
4.2. Effect of the self-sensing circuit on the mechanical response estimation

Next, it is examined how the self-sensing circuit dynamics affect the mechanical response estimation. When the mechanical response is extracted using the proposed self-sensing scheme, it is assumed that the scaling factor is constant with respect to frequency. This assumption was examined experimentally and the corresponding result was analyzed theoretically.

4.2.1. Experimental set-up. Another experiment was designed to specifically investigate the effect of the self-sensing circuit on the measured mechanical response. In the two different configurations shown in figure 6, ‘PZT A’ was used as a sensor and ‘PZT B’ as an actuator. The only difference between these two configurations is that the output voltage measured from PZT A is connected to the self-sensing circuit in figure 6(b). Therefore, by comparing the output voltage from the two configurations, the effect of the self-sensing circuit dynamics on the measured mechanical response could be examined. If the only role of the self-sensing circuit is amplification as assumed in equation (23), the output voltage measured from the second configuration should be simply a scaled version of the mechanical response measured from the first configuration. Otherwise, there exist additional signal distortions due to the self-sensing circuit.

4.2.2. Experimental results. Figure 7 shows the output voltages obtained from the above two configurations in the time and frequency domains. To examine the presence of the self-sensing circuit’s nonlinear behavior, it is examined if the output voltage from the second configuration can be scaled to get the mechanical response measured from the first configuration. In an ideal situation, the shape of the two output voltages should be identical except for scaling. However, there were additional distortions introduced by the self-sensing circuit. For instance, the output signal from the second configuration lagged behind that from the first configuration especially near one of the beam’s resonance frequencies. It is also revealed that the scaling factor is dependent on the driving frequency. To quantify the difference between these two output signals, the relative RMS error was calculated in a way similar to equation (25). The relative error value was 0.7402. The largest signal difference was observed near one of the beam’s resonance frequencies where the mechanical response reached its maximum amplitude (see figure 7). To eliminate the coupling effect of the PZT wafer with the host structure and isolate the source of the error, the self-sensing circuit was first examined using a conventional capacitor instead of the PZT wafer. It is shown that the scaling factor was constant with respect to the driving frequency when the conventional capacitor was employed. Therefore, it is concluded that the PZT capacitance value fluctuates with respect to the driving frequency due to the coupling effect of the PZT wafer with the vibration of the host structure. However, the frequency spectra in figures 7(c) and (d) show that the resonance frequencies of the target structure are successfully detected using the proposed self-sensing circuit.

4.1.2. Experimental results. Table 4 shows the errors between the scaling factor measured from the LCR meter and the ones estimated from three different algorithms when a ±4 V 10 kHz sine wave is applied as the input signal in step I. The trend of the experimental results is similar to that of the theoretical error analysis. That is, the orthogonality algorithm produces the smallest error, outperforming the RMS and LMS algorithms. Note that because the exact capacitance value is unknown in real experiments, the one measured from the LCR meter is used as the reference and its value is −0.826 44.

Table 4. Comparison of the three different self-sensing algorithms through experimental tests.

<table>
<thead>
<tr>
<th>Estimation algorithm</th>
<th>$\hat{SF}$</th>
<th>Relative error (%) as defined in equation (22)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS algorithm</td>
<td>−0.827 45</td>
<td>0.122</td>
</tr>
<tr>
<td>LMS algorithm</td>
<td>−0.827 39</td>
<td>0.115</td>
</tr>
<tr>
<td>Orthogonality algorithm</td>
<td>−0.826 77</td>
<td>0.041</td>
</tr>
</tbody>
</table>

* Note that $\hat{SF}$ is the scaling factor estimated from the self-sensing circuit. The $SF$ value estimated from the commercial LCR meter is used as the reference and its value is −0.826 44.

In step II, a ±4 V low-frequency chirp signal was used as the input signal. Figures 5(a) and (b) show the input chirp signal in the time and frequency domains. In figure 5(c), the output signal from the self-sensing circuit is shown. The contribution of the mechanical response is visible in the time domain, and it becomes more visible in the frequency domain, as shown in figure 5(d). Next, the scaling factor estimated from the LCR meter is perturbed to examine the effect of the scaling factor on the extracted mechanical response. Note that the qualitative characteristics of the time response shown in figure 5(e) is similar to the one obtained from the previous numerical simulation in figure 3(e). The resonance frequencies of the test structure were clearly identified from the extracted mechanical response as shown in figure 5(f). Similar to the simulation results in section 3, the error in the scaling factor estimate shifted the anti-resonance frequencies of the system horizontally. However, the resonance frequencies of the system were estimated properly.
Figure 5. Estimated mechanical responses in the time and frequency domains when a chirp signal (0–400 Hz) is applied and the error level of the scaling factor is varied ±2.5%. (a) A chirp input signal (0–400 Hz) in the time domain. (b) The frequency content of the chirp signal. (c) The measured output voltage in the time domain. (d) The frequency spectrum of the output voltage. (e) The extracted mechanical time response. (f) The frequency spectrum of the extracted mechanical response.

Figure 6. An experimental set-up for studying the effect of the self-sensing circuit on the extracted mechanical response: an input signal is applied to PZT B and the corresponding response is measured at PZT A with/without the self-sensing circuit for comparison. (a) Without the self-sensing circuit. (b) With the self-sensing circuit.
The effect of the mechanical response of the structure on the PZT capacitance is further investigated. The admittance of the PZT wafer attached to the structure is described as [19]

\[
Y(\omega) = \frac{b l}{i \omega} \left( \varepsilon_{33}^{T} - d_{31}^{2} Y_P \right) + \frac{Z_s(\omega)}{Z_s(\omega) + Z_a(\omega)} d_{31}^{2} Y_P \left( \frac{\tan \gamma l}{\gamma l} \right) \tag{26}
\]

where \(Z_s(\omega)\) is the mechanical impedance of the PZT wafer, \(Z_a(\omega)\) is the mechanical impedance of the structure, and \(\gamma\) is the wavenumber. If the PZT wafer is assumed to be a pure capacitor, the PZT capacitance value can be estimated from equation (26):

\[
C_p(\omega) = \frac{b l}{i \omega} \left( \varepsilon_{33}^{T} - \text{Re} \left\{ \frac{Z_s(\omega)}{Z_s(\omega) + Z_a(\omega)} \right\} \cdot d_{31}^{2} Y_P \right) \tag{27}
\]

where \(\tan(\gamma l)/\gamma l\) is assumed to be close to 1 in the frequency range used for our application [19]. \(\text{Re}[\cdot]\) denotes the real part of the complex number. The mechanical impedance, \(Z_s(\omega)\), is defined as the ratio of the applied force to the resulting velocity. Therefore, at the resonance frequencies of the structure, its mechanical impedance becomes very small. At other frequencies, the mechanical impedance of the structure is assumed to be much larger than that of the PZT wafer. Therefore, the PZT capacitance value fluctuates with respect to the driving frequency especially near the resonance frequencies of the structure. It is expected that there are additional errors introduced in the extracted mechanical response due to this fluctuation of the PZT capacitance.

4.3. Effect of the scaling factor estimate error on the mechanical response estimation

Here, it is examined how the error in the estimated scaling factor affects the accuracy of the extracted mechanical response based on equation (25). Because the true mechanical response is unknown in real experiments, the exact mechanical response term in equation (25) is approximated by attaching an additional PZT wafer collated with the PZT used for self-sensing. First, it is examined if the response of the collated PZT wafer is a good approximation of the mechanical response of the self-sensing PZT wafer, and the error analysis is performed to examine the effect of the scaling factor error.

4.3.1. Experimental set-up. Another experiment was designed to specifically investigate the effect of the scaling factor estimate error on the estimated mechanical response. Two different configurations were examined as shown in figure 8. In figure 8(a), a single PZT wafer was attached near the fixed end of the beam for exciting the beam and measuring the corresponding mechanical response. In figure 8(b), an additional PZT wafer, collocated with the existing PZT wafer,
was attached on the other side of the cantilever beam. In this second configuration, an input signal was applied to PZT A and the corresponding response was measured at PZT B with the self-sensing circuit. The second configuration was designed so that the response from PZT B can be used as an approximate of the true mechanical response expected at PZT A. Then, the mechanical response extracted from the first configuration was compared with that from the second configuration to examine the performance of the proposed self-sensing scheme. Similar to the numerical simulation, the error level in the scaling factor estimate was varied and their effects on the measured mechanical response were examined.

4.3.2. Experimental results. Figure 9 compares the mechanical responses obtained from the two configurations shown in figure 8 in the time and frequency domains. Using equation (25), it was examined how close the mechanical responses estimated from the first configuration were to the actual mechanical response obtained from the second configuration. The estimated relative RMS error was 0.5014, and the largest error was observed near one of the resonance frequencies (the fourth resonance frequency). It should be noted that two assumptions were made during the calculation of this relative RMS error. First, the exact PZT capacitance value in equation (22) was approximated by the one estimated by the LCR meter. Second, the PZT B response from figure 8(b) was assumed to be a good estimate of the true mechanical response term $v_p[k]$ in equation (25).

The large RMS error might be partially attributed to these assumptions. For instance, the PZT B response would be inherently different from that of PZT A for several reasons. First, the bonding conditions of the two PZT wafers to the beam and/or their capacitance values were most likely not identical. In figure 9, it was observed that the amplitude of the PZT B response was slightly larger than that of PZT A particularly near the fourth resonance frequency. This implied that PZT B might have had better coupling with the beam or a larger capacitance value than PZT A. Second, it is possible that two PZT wafers were not exactly collocated nor had the exact same size. In fact, PZT A was placed on the top surface of the beam, and PZT B was attached on the bottom surface. This could have created additional errors. In spite of these constraints, the shapes of the mechanical responses resulted from two configurations were qualitatively similar.

The frequency spectra in figures 9(c) and (d) also demonstrate the effectiveness of extracting the resonance frequencies of the target structure. This suggests that the proposed self-sensing scheme functioned well, as expected from the theoretical analysis and numerical simulations.

5. Conclusion

In this study, a combination of self-sensing algorithms and a self-sensing circuit is developed particularly for active sensing devices. The goal of the proposed self-sensing is to use a single active sensing device such as a PZT transducer for simultaneous excitation and sensing. The analytical model that describes the interactions between the PZT wafer and the host structure is developed, and a circuit analysis is performed to describe the relation between the PZT input and output voltages and the corresponding mechanical response voltage. It is shown that the amplitude ratio of the PZT output voltage to the PZT input voltage can be approximated by a so-called scaling factor, which is defined as the ratio of the PZT capacitance value to the feedback capacitor’s capacitance value in the self-sensing circuit. In the first step, a specific probing input signal is applied to the PZT wafer to calibrate the self-sensing algorithm and to estimate the scaling factor. Then, the mechanical time response corresponding to an arbitrary input signal is extracted in the second step using the previously calibrated self-sensing algorithm. The feasibility of the proposed self-sensing scheme is demonstrated through numerical simulations and experimental tests. In most cases, the estimation error of the scaling factors was less than 0.5% and the mechanical response of the system was successfully extracted in the time and frequency domains. In contrast to the previous self-sensing studies, which are mainly designed for vibration control applications, the proposed self-sensing scheme emphasizes the accurate extraction of the mechanical response in the time domain. The proposed self-sensing scheme is straightforward, and its adaptive nature makes it attractive for field applications where the system is subjected to varying operational and environmental conditions. Ongoing research is underway to take advantage of the proposed self-sensing for the development of PZT transducer self-diagnosis and structural damage diagnosis schemes.
Active self-sensing scheme development for structural health monitoring

Figure 9. The output responses and the frequency spectrum obtained from the two different configurations shown in figure 8 are compared to investigate the performance of the proposed self-sensing scheme. (a) Mechanical response with one PZT wafer and the self-sensing scheme. (b) Mechanical response with two collocated PZT wafers. (c) Frequency spectrum with one PZT wafer and the self-sensing scheme. (d) Frequency spectrum with two collocated PZT wafers.

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References