In situ measurement of structural mass, stiffness, and damping using a reaction force actuator and a laser Doppler vibrometer
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Abstract
The problem of structural parameter measurements needs a total solution bridging theory and experiments. In this paper, a practical methodology for in situ measurements of structural mass, stiffness, and damping is presented for built-up structures. As for the experimental uniqueness of the methodology, a reaction force actuator and non-contact optical device are utilized respectively as an input force generator and output displacement measurer, providing a fundamental data set of the proposed numerical algorithm for data-driven structural parameter estimation. The algorithm autonomously estimates the diagonalized mass, symmetric stiffness, optimal non-proportional damping, and suboptimal proportional damping matrices for multi-degrees-of-freedom structures. Structural parameter measurements of two built-up structures followed by a comparison with conventional measurements are used as examples for verification of the accuracy of the proposed methodology.

1. Introduction
Over the past few decades, momentous practical achievements in modal parameter estimation for built-up structures both in the laboratory and field have been reported for measurements of structural dynamic characteristics [6, 16]. The estimated modal parameters (i.e., natural frequency, mode shape, and modal damping) play a pivotal role in understanding tested structures and developing accurate finite element models. While great advances have been made in modal parameter estimation, comparatively less research has been aimed at directly obtaining structural parameters (i.e., spatial distributions of mass, stiffness, and damping). Direct structural parameter estimation from in situ dynamic testing requires more sophisticated methodologies, both experimentally and theoretically, which distinguishes itself from classical modal parameter estimation. The methodologies are delineated as solving technological challenges categorized mainly into two groups: (1) hardware problems regarding actuation and sensing equipment and (2) theoretical problems regarding data processing algorithms.

Forced vibration tests need high capacity actuators. However, exciting large-scale structures in a controlled and repeatable manner still remains challenging due to the considerable costs and efforts: direct attachment of the moving portion of an actuator to a tested structure requires a firm foundation or retaining wall on which the actuator body is fixed. Thus, direct actuation is rather appropriate for laboratory structures. Meanwhile, reaction force actuators (RFAs) are preferred for dynamic testing in the field. A moving portion of the actuator works as a reaction mass and thus generates an inertial force transmitted to the actuator body.
Structural parameter estimation is an inverse problem of identifying the best-fit (i.e., optimal) physical model from measured input and output data. Various developments of numerical algorithms are reported in the literature: (1) time domain estimation of parameterized finite element models [8, 10]; (2) conversion of estimated transfer functions to structural parameters [18, 2]; (3) extraction of structural parameters from generalized eigenvalue problems using a known added mass [5, 4]; (4) conversion of estimated state-space models to structural parameters [1, 23, 13]. One of the technological challenges commonly issued in the majority of the studies is incomplete mass estimation, which results in the need of a priori partial or full information of the structural mass for the structural parameter estimation. In addition, many of the studies are confined to numerical examples without extensive experimental verifications.

Among the four approaches, state-space model-based structural parameter estimation has gained much attention due to the orthodox theories in the fields of system, control, and identification [15, 9]. In particular, subspace system identification is considered a breakthrough in identification and provides the optimal state-space model estimated from measured input and output data [21, 22]. A theoretical condition for complete conversion from a state-space model to physical parameters is proposed: physical quantities of input and output measurements shall be force and displacement, respectively [1, 13]. Under the condition, the structural mass can be reconstructed from the identified state-space model [13]. However, a practical methodology for in situ structural parameter measurements has remained uncovered so far.

In this study, the problem of in situ structural parameter measurements is tackled: a novel integrated methodology overarchin g actuation, measurement, and data processing is proposed for a theoretical and experimental solution of in situ structural parameter measurements. An RFA is strategically adopted as a cost-effective actuation source to built-up structures. A non-contact optical device using a laser Doppler vibrometer (LDV) is proposed as a practical solution for a precision displacement measurement of vibratory structures [19]. Most importantly, a unique algorithm of state-space model-based structural parameter estimation from a data set of the measured moving mass acceleration of an installed RFA and structural displacement responses is developed based on known information of the moving mass of the RFA. The high accuracy of the structural parameter measurements is proved through two experimental examples of a two-story building and a large-scale loading frame.

2. Theoretical development of in situ structural parameter measurements

The ultimate goal of system identification in structural engineering is constructing the mathematically optimal but structurally interpretable model from experimental data—the structural identification problem aims at measurements of structural parameters. In this context, state-space model-based structural parameter estimation was numerically studied at an early stage [17, 1]. Very recently, theoretical and experimental work on structural parameter estimation from an identified state-space model for earthquake-induced vibratory structures has been reported [13] which lays down the theoretical basis of this study. Based on the previous achievement of structural stiffness estimation from structural acceleration responses and partial information of the structural mass, a theoretical formulation tailored to in situ measurements for structural mass, stiffness, and damping using a modal shaker is presented in this section.

2.1. Problem statement

The dynamic motion of an n-story building (figure 1) under ith floor excitation by a modal shaker with a movable portion of known mass, \( \bar{m}_i \), is described as

\[
M \ddot{d}_i(t) + C \dot{d}_i(t) + K d_i(t) = -\bar{m}_i u(t)
\]

where \( M, C, K \in \mathbb{R}^{n \times n} \) are mass, damping, and stiffness matrices, respectively; \( \dot{d}_i(t), \ddot{d}_i(t), \bar{d}_i(t) \in \mathbb{R}^n \) are vectors
of each floor's displacement, velocity, and acceleration, respectively; \([-\bar{m}_i] \in \mathbb{R}^n\) is a vector which has elements of \(-\bar{m}_i\) at the \(i\)th row and zeros elsewhere; \(u(t)\) is the exciting acceleration to the moving mass. Considering displacement measurement of each floor's response, a continuous-time state-space model for the linear dynamical system is formulated as

\[
\dot{x}(t) = A_{e} x(t) + B_{e} u(t)
\]

\[
y(t) = C_{e} x(t)
\]

where the state vector is defined as \(x(t) = \{d(t)^T \ d(t)^T\}^T\) and the linear system matrices are determined as

\[
A_{e} = \begin{bmatrix} 0 \\ -M^{-1}K - M^{-1}C \end{bmatrix} \in \mathbb{R}^{2n \times 2n}
\]

\[
B_{e} = \begin{bmatrix} [0] \\ [-\bar{m}/m] \end{bmatrix} \in \mathbb{R}^{2n}
\]

\[
C_{e} = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{n \times 2n}
\]

where \(m_i\) is the sum of the lumped mass at the \(i\)th degree-of-freedom of the building structure and the stationary mass of the modal shaker placed on the \(i\)th floor hereinafter, this is referred to as 'loaded mass' (figure 1) in this paper; \([-\bar{m}/m] \in \mathbb{R}^n\) is a vector which has elements of \(-\bar{m}/m_i\) at the \(i\)th row and zeros elsewhere; \([0] \in \mathbb{R}^n\) is a null vector. Now, the problem of \textit{in situ} structural parameter measurement can be stated as estimation of the unknown structural parameters in equation (4) through experimental data processing.

2.2. Experimental data processing

The theory of subspace system identification for realization of a minimal state-space model from arbitrary input and output data is herein revisited, focusing on the numerical algorithms for subspace state-space system identification (N4SID) [13]. The block Hankel matrix of system input is constructed from the \(2i + j\) measured discrete data and is partitioned into past (i.e., from 0 to \(i - 1\)) and future (i.e., from \(i\) to \(2i - 1\)) blocks:

\[
U_{0[2i-1]} = \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ \vdots & \ddots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \\ \vdots & \ddots & \ddots & \vdots \\ u_{2i-1} & u_{2j} & \cdots & u_{2i+j-2} \end{bmatrix} = \begin{bmatrix} U_{0[i-1]} \\ U_{i[2i-1]} \end{bmatrix}
\]

Similarly, the output block Hankel matrix, \(Y_{0[2i-1]}\), is built from the \(2i + j\) measured discrete data and partitioned into past and future output blocks. By defining the joint space of the past input and past output as \(W_p = [U_p \ Y_p]^T\), an oblique projection of the row space of \(W_p\) along the row space of \(U_I\) can be calculated and denoted as \(P_i\). This oblique projection is numerically calculated by LQ decomposition of the block Hankel matrices and is equal to the production of the extended observability matrix and state sequence as:

\[
P_i = O_iX_i.
\]

Singular value decomposition of the projection, \(P_i\), leads to [20]:

\[
P_i \cong \begin{bmatrix} F_1 & F_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_1^T \\ G_2^T \end{bmatrix} = F_1S_1G_1^T
\]

where \(S_1\) is the diagonal matrix of positive singular values; \(F_1\) and \(G_1\) are the matrices whose columns are orthogonal to each other and correspond to \(S_1\); \(F_2\) and \(G_2\) are the matrices whose columns are orthogonal to each other and correspond to zero singular values. By splitting the results of singular value decomposition, the extended observability matrix and the state sequence can be extracted from the decomposed projection matrix, respectively as:

\[
O_i = F_1S_1^{1/2}; \quad X_i = S_1^{1/2}G_1^T.
\]

Another oblique projection termed \(P_{i-1}\) is calculated from the one-step shifted input and output block Hankel matrices and then the one-step shifted state sequence is calculated as:

\[
X_{i+1} = (O_{i-1})^+P_{i-1}
\]

where \(O_{i-1}\) is equivalent to \(O_i\) with the last block row omitted and \(^+\) is the pseudo-inverse operator. Finally, a least squares solution of the system matrices (i.e., realization of a discrete-time state-space model) can be calculated by a pseudo-inverse [20]:

\[
\begin{bmatrix} A_d \\ C_d \end{bmatrix} = \begin{bmatrix} X_{i+1} \\ Y_{ii} \end{bmatrix} ^+ \begin{bmatrix} X_i \\ U_{ii} \end{bmatrix}
\]

In the specific experiment (figure 1) of an \(n\)-story building structure excited by a modal shaker, the system input (i.e., the exciting acceleration of the shaker moving mass) is not simultaneously transmitted to the system output (i.e., the measured displacement responses of each floor)—technically speaking, the dynamic system of the vibratory structure excited by a modal shaker is proper. Thus, the feed through matrix \(D_d\) should be set equal to null and is omitted hereinafter in the paper. The discrete-time state-space model in equation (12) is converted into a continuous-time domain model:

\[
A_c = \frac{1}{\Delta t} \ln(A_d)
\]

\[
B_c = \left( \int_0^{\Delta t} \exp(A_{c}\tau)d\tau \right)^{-1} B_d
\]

\[
C_c = C_d
\]

where \(\Delta t\) is the sampling time of the measured data. Since the estimated system matrices result from a specific realization with arbitrary state coordinates, conversion to
The problem of structural parameter estimation in this study is stated as reconstructing mass, stiffness, and damping matrices from the identified state-space model and the given moving mass of a modal shaker. First, the loaded mass, \( m_i \), is calculated by setting the \( i \)-th elements of \([Z]\) in equation (19), \( z_i \) equal to \(-\bar{m}/m_i\). Namely, \begin{equation}
m_i = -\bar{m}/z_i.
\end{equation}

The remaining structural parameters are optimally estimated by solving a linear regression problem with \textit{a priori} information of the known loaded mass. Composing a linear regression problem starts with the following identities between equations (4) and (18) as \begin{align}
V &= -M^{-1}K 
\end{align}
and \begin{align} W &= -M^{-1}C. \end{align}
In order to deal with the complex matrix algebra efficiently, vectorized expression of matrix [3] is adopted in this study. By pre-multiplying mass matrix to both sides of equation (22) and stacking each column of the resulting matrix, the equation leads to: \begin{equation}
\text{vec}(MV) = -\text{vec}(K) \in \mathbb{R}^{n^2 \times 1}
\end{equation}
where \text{vec}(\bullet) denotes the vectorized expression. Considering the Kronecker product, \( \otimes \), it is stated: \begin{align}
\text{vec}(MV) &= [V^T \otimes I]\text{vec}(M) = -\text{vec}(K) \quad (25)
\end{align}
\begin{align}
\text{vec}(MW) &= [W^T \otimes I]\text{vec}(M) = -\text{vec}(C). \quad (26)
\end{align}
By combining equations (25) and (26), a linear equation for the vectorized structural parameters is built as:
\begin{equation}
\begin{bmatrix}
[V^T \otimes I] & 0 & I \\
[W^T \otimes I] & I & 0
\end{bmatrix}
\begin{bmatrix}
\text{vec}(M) \\
\text{vec}(C) \\
\text{vec}(K)
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (27)
\end{equation}
To uniquely determine the optimal structural parameters by imposing constraints to equation (27), two assumptions are made in this study: (1) symmetricity is valid for the stiffness matrix; (2) the mass matrix is diagonalized. The assumption of the symmetric stiffness matrix (i.e., \( K = K^T \)) yields another identity equation from equation (22) as \begin{equation}
MV = V^TM. \quad (28)
\end{equation}
Equation (28) ends up with an equation for the vectorized mass as \begin{equation}
[V^T \otimes I]\text{vec}(M) = [I \otimes V^T]\text{vec}(M). \quad (29)
\end{equation}
Adding the symmetric constraint of equation (29) to equation (27) yields \begin{equation}
\begin{bmatrix}
[V^T \otimes I] & 0 & I \\
[W^T \otimes I] & I & 0 \\
[V^T \otimes I] - [I \otimes V^T] & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\text{vec}(M) \\
\text{vec}(C) \\
\text{vec}(K)
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (30)
\end{equation}
Based on the assumption of the diagonalized mass matrix, the problem of structural parameter reconstruction is further simplified: \( n^2 - n + 1 \) elements of the mass matrix including \( n^2 - n \) zeros of off-diagonals and a non-zero diagonal (i.e., the known loaded mass) are utilized to reconstruct the other structural parameters. equation (30) can be partitioned after pivoting:
\begin{equation}
\begin{bmatrix}
P_M & P_{MKC}
\end{bmatrix}
\begin{bmatrix}
\text{vec}(M_{\text{given}}) \\
\text{vec}(M_{\text{others}}, C, K)
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (31)
\end{equation}
where \( \text{vec}(M_{\text{given}}) \in \mathbb{R}^{n^2-n+1} \) is the given elements of the mass matrix and \( P_M \in \mathbb{R}^{3n^2 \times n^2-n+1} \) is the column-wise pivoted regressor matrix corresponding to the given mass; \( \text{vec}(M_{\text{others}}, C, K) \in \mathbb{R}^{2n^2-n} \) is the other structural parameters and \( P_{\text{MCK}} \in \mathbb{R}^{3n^2 \times 2n^2+n-1} \) corresponds to another regressor matrix. A least squares solution for the to-be-determined parameter vector can be calculated by the pseudo-inverse as:

\[
\text{vec}(M_{\text{others}}, C, K) = P_{\text{MCK}}^{-1}(-P_M \text{vec}(M_{\text{given}})). \tag{32}
\]

Finally, structural parameters of a multi-degrees-of-freedom system are reconstructed in the matrix forms through unstacking the vectors of \( \text{vec}(M_{\text{given}}) \) and \( \text{vec}(M_{\text{others}}, C, K) \). It should be noted that the reconstructed structural mass, stiffness, and damping are the optimal estimates, since they are the least squares solutions converted from the optimal estimate of state-space models; hereinafter, the subscript ‘opt’ is added to the structural parameter estimates.

The classical assumption of damping proportional to mass and stiffness (i.e., Rayleigh damping) yields a symmetric damping matrix under the condition of symmetric mass and stiffness matrices. Considering proportional damping to diagonalized mass and symmetric stiffness, the symmetric assumption of damping is identical with that of stiffness. Thus, the system metric damping assumption is not valid as an independent constraint to the regression problem, even if applied. In other words, the data-driven structural parameter estimation proposed in this paper does not involve the symmetric damping constraint in order to estimate the optimal damping. Thus, the damping estimate from equation (32) does not guarantee its symmetry.

**2.4. Suboptimal estimation of symmetric damping matrix**

As described in section 2.3, the damping estimate in equation (32) is generally non-symmetric but optimal to the given experimental data. This can be interpreted with the concept of a gap (i.e., model discrepancy) between the physical and experimental models as being frequently issued in the field of system identification [7, 13]. The physical interpretation of the optimal damping estimate is challenging due to its lack of symmetry. Meanwhile, a symmetric damping model has been popularly adopted in the modal testing and structural engineering communities for the purpose of building a analytical and computational damping model with physical assumptions. In this context, a methodology of symmetric damping estimate is proposed by balancing both mathematical optimality and physical symmetricity and termed suboptimal proportional damping in this study.

For a realization of proportional damping, the model of Rayleigh damping is taken by a linear combination of the optimal mass and stiffness estimates:

\[
C_{\text{prop}} = \alpha_0 M_{\text{opt}} + \alpha_1 K_{\text{opt}}. \tag{33}
\]

An optimization problem for finding the suboptimal proportional damping is composed using the given optimal estimates of mass, stiffness, and damping matrices from equation (32) as:

\[
\{\hat{\alpha}_0, \hat{\alpha}_1\} = \arg \min_{\{\alpha_0, \alpha_1\}} \| C_{\text{opt}} - C_{\text{prop}} \|^2 = \arg \min_{\{\alpha_0, \alpha_1\}} \| C_{\text{opt}} - (\alpha_0 M_{\text{opt}} + \alpha_1 K_{\text{opt}}) \|^2. \tag{34}
\]

An element-wise expression of the cost function in equation (34) leads to:

\[
\{\hat{\alpha}_0, \hat{\alpha}_1\} = \arg \min_{\{\alpha_0, \alpha_1\}} \sum_{i,j} (c_{ij} - (\alpha_0 m_{ij} + \alpha_1 k_{ij}))^2 \tag{35}
\]

where \( m_{ij}, k_{ij}, \) and \( c_{ij} \) are the \((i,j)\)th elements of the estimates of mass, stiffness, and damping matrices, respectively. By setting the derivatives of the cost function in equation (35) with respect to each variable (i.e., \( \alpha_0 \) and \( \alpha_1 \)) equal to zero, respectively, a pair of linear equations for the two variables are composed. Eventually, the solution is calculated at the intersection point of the two linear equations. The mathematical process is geometically explained in figure 2. Once the proportionality constants (i.e., \( \alpha_0 \) and \( \alpha_1 \)) are estimated, the proportional damping is easily calculated from equation (33).

**3. Experimental verification of in situ structural parameter measurements**

In this section, two experimental examples are presented to demonstrate the developed theory for in situ structural parameter measurements: first, a dynamic test with a laboratory-scale two-story building and its estimated structural parameters are discussed in detail with comparison to conventional measurements. Next, a large-scale loading frame is introduced as a more realistic example of built-up structures.

**3.1. Example 1—a two-story steel frame building**

The test structure is a two-story single-bay steel frame building with 0.5 m story heights (figure 3(a)). Each floor is
constructed as a rigid diaphragm (80 cm × 60 cm × 1 cm thick) spot-welded along all four edges to 4 cm × 3 cm rectangular beams. Each floor is supported by four steel rectangular (100 cm × 10 cm × 0.6 cm thick) plates acting as columns—bolted beam–column connections are adopted. Each column is erected on a base plate attached to a rigid foundation by two anchor bolts. Prior to building the structure, each structural element was weighed: the masses of a floor and a column were 61.8 and 4.5 kg, respectively—the total of weight of the structure was 149.2 kg including the base plates.

A commercial RFA (400 ELECTRO-SEIS shaker from APS Dynamics) was utilized to excite the structure in this study (figure 3(b)). The electro-dynamic modal shaker consists of movable portion (i.e., armature and attached mass blocks) and a stationary portion (i.e., shaker body). Among various operational modes, a free armature mode allows us to use the shaker as an RFA. The operating frequency range is DC to 200 Hz, the maximum reaction force is 445 N (up to 20 Hz), and the maximum stroke is 158 mm.

An LDV (PSV-400 from Polytec GmbH) was adopted to measure dynamic displacements of the vibratory structure (figure 3(c)). The LDV is a precision optical transducer to determine the vibration velocity and displacement by sensing the frequency shift of back scattered light from a moving object. A scanner module attached at the LDV head enables it to aim a laser beam at multiple points of the structures. Since the LDV is a non-contact measurement system, in situ measurement of built-up structures is convenient just enough to stand itself away from the structures.

In the experimental phase of this study, the shaker was horizontally placed on the first floor of the test structure, generating a 2–15 Hz chirp motion. To investigate the accuracies of structural parameter estimates under various situations, four different tests were conducted with varying experimental configurations: CASE II shared the identical experimental setup to CASE I (figure 4(a)) except with a decreased driving voltage to the shaker of half of that in CASE I; CASE III (figure 4(b)) and CASE IV (figure 4(c)) have one and two additional mass block(s) on the second floor, respectively. In all tests, the motion of the moving mass was sensed by an accelerometer. Structural responses at the midpoints of each floor’s rectangular beam (i.e., red circles in figure 3(a)) were monitored by a remote LDV located 5 m away from the structure while aligning along the planar axis of vibrating direction of the structure. 32 s data were collected with a sampling rate of 256 samples s⁻¹.

Using the measured data of moving mass acceleration and each floor’s displacement in CASE I (shown in figure 5), the structural parameters of the test structure were estimated by the proposed methodology with the given information of

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Figure 3. (a) Isometric view of the tested structure—the aiming points of a laser beam from LDV are highlighted with red circles; (b) electro-dynamic modal shaker; (c) laser Doppler vibrometer.

Figure 4. Experimental configurations: (a) CASE I and CASE II; (b) CASE III: one additional mass block on the second floor; (c) CASE IV: two additional mass blocks on the second floor.
Table 1. Estimates of mass and inter-story stiffness (units: kg and N m$^{-1}$).

<table>
<thead>
<tr>
<th></th>
<th>CASE I</th>
<th>CASE II</th>
<th>CASE III</th>
<th>CASE IV</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$ (loaded mass)</td>
<td>154.2</td>
<td>154.3</td>
<td>154.1</td>
<td>154.4</td>
<td>154.3</td>
</tr>
<tr>
<td>$m_2$</td>
<td>66.1</td>
<td>66.3</td>
<td>72.4</td>
<td>78.7</td>
<td></td>
</tr>
<tr>
<td>$k_1$</td>
<td>220.525</td>
<td>220.718</td>
<td>219.974</td>
<td>220.450</td>
<td>220.417</td>
</tr>
<tr>
<td>$k_2$</td>
<td>149.176</td>
<td>149.366</td>
<td>149.355</td>
<td>149.405</td>
<td>149.326</td>
</tr>
</tbody>
</table>

11.3 kg moving mass as:

$$\hat{M}_{\text{opt}} = \begin{bmatrix} 154.2 & 0 \\ 0 & 66.1 \end{bmatrix};$$

$$\hat{K}_{\text{opt}} = \begin{bmatrix} 369.701 & -150.759 \\ -150.759 & 147.592 \end{bmatrix};$$

$$\hat{C}_{\text{opt}} = \begin{bmatrix} 104.8 & -46.9 \\ -3.0 & 6.0 \end{bmatrix};$$

where the units of mass, stiffness and damping matrices are kg, N m$^{-1}$, and N s m$^{-1}$, respectively. Story masses and inter-story stiffnesses were then calculated from the estimated mass and stiffness and are summarized in Table 1, including those from the other cases.

The loaded masses (i.e., mass regarding the first floor) were estimated consistently in all cases, yielding a mean of 154.3 kg, which is equivalent to the sum of the floor mass, stationary portion mass of the shaker and 80.5% of the four columns’ mass (i.e., $61.8 + 78 + 4.5 \times 4 \times 0.805 = 154.3$). The mass regarding the second floor was estimated as 66.2 kg (the mean of the estimated masses in CASE I and CASE II) which is equivalent to the sum of floor, stationary portion of the shaker and 24.5% of the four columns’ mass (i.e., $61.8 + 4.5 \times 4 \times 0.245 = 66.2$). The incremental masses resulting from the additional mass block on the second floor were estimated as 6.2 kg (=72.4 – 66.2) and 6.3 kg (=78.7 – 72.4), respectively, for CASE III and CASE IV. The increments were close to the measured block mass of 6.1 kg. Hence, the high precision of mass measurement by the proposed methodology was proved. It is worth noting that the approach of adding a known mass to the test structures is also utilized for a hardware system calibration of the proposed methodology—compensation of errors relating to alignment of shakers, sensitivity of accelerometers, laser beam aiming angle of LDVs, etc.

A static test was carried out to cross-check the structural stiffness: horizontal static deflection was induced by pushing the second floor and then the pushing force and corresponding deflection were measured by a strain gage type force sensor and a vernier caliper, respectively. The estimated structural stiffness from the static test was 87.8 N m$^{-1}$. Based on the mean of inter-story stiffnesses in Table 1, the structural stiffness was calculated as 89.0 N m$^{-1}$ (= $(k_1 \times k_2)/(k_1 + k_2)$) having a 1.4% discrepancy with respect to the result from the static test. Hence, the high precision of the stiffness measurement by the proposed methodology was proved.

Table 2 summarizes three types of damping estimates for all cases: (1) the optimal and (2) suboptimal proportional damping estimates (equations (32) and (35), respectively) by the proposed methodology and (3) proportional damping by the conventional half-power method using frequency response functions. The optimal damping estimates are non-symmetric—namely, a pair of off-diagonals are different. Furthermore, the off-diagonals are less consistent or rather arbitrary in each case. In contrast, the proportional damping estimates are symmetric and even very consistent in all cases—this means the proportional damping estimate is less varying within the specific ranges of the structural changes.
The corresponding rms average errors are 2
model of the estimated mass, stiffness, and optimal damping.
measurement in CASE I and the prediction from the structural
figure 6 displays comparison displacement plots between the
non-symmetric damping estimate is given in figures 6 and 7:
of vibration amplitude and mass. Proof of optimality of the
non-symmetric damping estimate is given in figures 6 and 7:
suboptimal proportional damping estimate—increased rms
average errors are found as 8
Meanwhile, figure 7 is a prediction from the model with the
supporting columns are erected with a 3.5 m center-to-center
distance. An I-shape (30 cm wide and 70 cm in depth) cross
beam connects the two columns, being 1.6 m away from the
top of the columns. The shaker (but with 18.4 kg reaction
mass) used previously was again mounted on the middle of the
cross beam aligning the actuation direction to the planar axis
of the structure (figure 8(a)). The LDV was set 5.8 m apart
from the left column, aiming the laser beam at the level of the
cross beam—hence, giving a 23.9◦ angle. A similar experiment to the previous
one for the two-story building was repeated with an excitation
displacement measurement is corrected off-line by dividing
it by cosine of the angle. A similar experiment to the previous
one for the two-story building was repeated with an excitation
frequency range of 2–25 Hz and an increased driving voltage
to the shaker.

Figures 9(a) and (b) display the measured moving
mass acceleration and structure’s displacement corrected with
the aiming angle, respectively. Considering the structure
as a single-degree-of-freedom system, structural parameters
were estimated by the proposed algorithm: the optimal
parameters were estimated as 2724 kg, 5.08 MN m−1,
and 1.34 kN s m−1, respectively, for mass, stiffness, and
damping. Since direct measurement of the structural mass and
stiffness was challenging, an alternative approach to obtain
Figure 7. Predicted displacements from estimated mass, stiffness, and suboptimal proportional damping: (a) comparison of measured and prediction of the second floor and (b) corresponding error; (c) comparison of measured and prediction of the first floor and (d) corresponding error.

Figure 8. Dynamic test of a loading frame: (a) actuation; (b) measurement.

The parameters was adopted to evaluate the accuracy of the estimates: based on the drawings of the structure, masses for structural elements were calculated—a cross beam (1607 kg) and a column (775 kg). A natural frequency of 6.872 Hz was identified from a Fourier transform of the free vibration responses in an impact hammer testing for the structure. Then, structural stiffness was estimated from the known mass and natural frequency. The mass estimate of 2724 kg is equivalent to the sum of the cross beam mass, stationary portion mass of the shaker and 67.0% of two columns’ mass (i.e., 1607 + 78 + 775 × 2 × 0.67 = 2724). The estimated mass and natural frequency yield the structural stiffness as 5.09 MN m⁻¹, which is very close to the stiffness estimate (i.e., 5.08 MN m⁻¹) from the proposed methodology. The damping estimate (i.e., 1.34 kN s m⁻¹) is equivalent to 0.6% of the critical damping and thus the structure is very lightly damped. Figure 9(c) displays the measured displacement with a superimposed prediction from the estimated structural parameters: the corresponding error is plotted in figure 9(d) and calculated as an rms average of 3.56 × 10⁻⁴ m.

4. Conclusions

In this study, a novel experimental methodology of structural parameter measurements for built-up structures was presented with an inclusive solution for equipment and algorithms. The methodology utilized a reaction force actuator and a remote optic device, respectively, for practical actuation and a precision displacement measurement of built-up structures. To efficiently process the experimental data from reaction force-induced vibratory structures, a rigorous autonomous algorithm was developed for structural parameter estimation. Based on the fundamental assumptions of the diagonalized mass and symmetric stiffness matrices, full information
Figure 9. Measured and predicted displacements of the controlled excitation test for the loading frame: (a) measured acceleration of the shaker moving mass; (b) corresponding displacement responses; (c) comparison of the measured and predicted displacement and (d) the corresponding error.

regarding the structural mass, stiffness, and damping was reconstructed from the identified state-space model. The key innovation of the structural parameter reconstruction was the use of known mass information on the moving portion of an installed reaction force actuator. Experimental verifications for a multi-story building and a large-scale loading frame structures revealed the high accuracy of the estimated structural parameters by the proposed methodology. In addition, a unique concept of suboptimal proportional damping was introduced and successfully demonstrated by yielding consistent structural damping under changes of structural mass and vibration amplitude.

Future work should be directed toward quantitative analyses on: (1) the column mass participating ratio to adjacent floor masses—columns with various flexural rigidities will be examined; (2) the effect of structural damage to structural parameters—in addition to the use of stiffness, the suboptimal proportional damping may provide a metric of structural damage; (3) the effect of environmental variables on structural parameters—a robust tool for structural integrity evaluation can discriminate environmental effects such as temperature change and adding mass by precipitation, etc. Most importantly, structural parameter estimation for full scale multi-DOFs structures is planned for further demonstration of the proposed methodology.

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