Fatigue crack localization using noncontact laser ultrasonics and state space attractors

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A fatigue crack and its precursor often serves as a source of nonlinear mechanism for ultrasonic waves, and the resulting nonlinear features are often much more sensitive to the fatigue crack than their linear counterparts. Among various nonlinear ultrasonic techniques, the proposed laser nonlinear wave modulation spectroscopy (LNWMS) is unique in that (1) it utilizes a pulse laser to exert a single broadband input instead of conventional two distinctive sinusoidal waves, and (2) a complete noncontact measurement can be realized based on LNWMS. Under a broadband excitation, a nonlinear source exhibits modulations due to interactions among various input frequency components. These modulations are often weak and can be hardly directly detected. In this paper, a damage feature called Bhattacharyya distance is extracted from the ultrasonic time signal corresponding to a pulse laser input and used to quantify the degree of damage-induced nonlinearity and localize the crack. This feature is a measure of a statistical distance used to detect the geometrical changes between state space attractors reconstructed before and after damage formation. It has been successfully used for localizing fatigue cracks in metallic plates.

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I. INTRODUCTION

As the failures caused by fatigue crack constitute nearly 90% of failures of in-service metallic structures, fatigue has been a critical concern for the reliability and performance of high expenditure, safety critical metallic structures.1 Structural health monitoring (SHM) and non-destructive testing (NDT) have been recognized as potential solutions to address these safety issues so that catastrophic failures can be prevented and maintenance costs can be minimized. Among various SHM and NDT techniques, linear ultrasonic techniques have been widely studied for detecting the presence of cracks.2–6 The conventional linear ultrasonic techniques detect a crack by measuring variations of the amplitude, phase and mode conversion of ultrasonic waves which are either transmitted or reflected from the crack. However, a fatigue crack typically grows rapidly and leads to a sudden catastrophic failure once it becomes visible and detectable by the conventional linear techniques. That is, the linear ultrasonic techniques are not sensitive enough to detect a fatigue crack at its early stage.

Recent studies have shown that a fatigue crack and its precursor often serve as a source for generating nonlinear waves and the sensitivity of the nonlinear ultrasonic techniques to the fatigue crack is much higher than what can be achieved by the conventional linear ultrasonic techniques.7–9 In the nonlinear ultrasonic techniques, a fatigue crack acts as an active nonlinear radiation source of new frequency components rather than a passive scatter as in the conventional linear techniques. This makes the nonlinear ultrasonic techniques a unique damage-sensitive instrument for localizing and imaging a nonlinear defect.

Nonlinearity due to crack formation manifests itself as distortion, accompanying wave harmonics, and in sum and difference frequency generation (sidebands). For linear intact structures, these nonlinear features are weak, but they become remarkably strong for damaged structures.10–14 Nonlinear wave modulation spectroscopy (NWMS) is one of the nonlinear ultrasonic techniques based on nonlinear mixing of two distinctive input signals.10 Normally, a low-frequency pumping input and a high-frequency probing input are used in NWMS to create modulation. Here, the amplitude of modulation heavily depends on the choice of the probing and pumping frequencies, and the optimal combination of these two input frequencies is also affected by environmental and operational conditions (e.g., temperature and loading) of the target structure and even by defect configurations.13,15–17 Therefore, finding the optimal combination of the probing and pumping frequencies, which can maximize the modulation level, becomes a moving target. To address this problem, the frequencies of either the probing signal17 or both the probing and pumping signals are swept.13 On the other hand, a laser nonlinear wave modulation spectroscopy (LNWMS) utilizes a laser pulse as an input signal instead of using two distinct frequency inputs.18 Because of a broadband nature of the laser pulse input, no clear modulation frequency can be

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identified. Instead, a fatigue crack is detected by counting the number of spectral peaks (Sideband Peak Count, SPC) above a moving threshold based on the premise that a spectral signal obtained from a nonlinear system would have more spectral peaks compared to a linear system. Experimental test results obtained from simple plates and aircraft fitting-lugs demonstrate that an increased number of spectral peaks appear with a fatigue crack.

In this study, a damage detection technique is developed so that a damage feature, which is sensitive to a nonlinear damage mechanism such as fatigue crack, can be extracted from the state space representation of the ultrasonic response obtained by LNWMS and used for locating and visualizing the crack. The proposed damage detection technique offers the following advantages: (1) A pulse excitation input is used rather two distinctive sinusoidal inputs for the extraction of nonlinear modulation responses; (2) a complete noncontact laser ultrasonic system is adopted for LNWMS measurement by integrating and synchronizing a Q-switched neodymium-doped yttrium aluminum garnet (Nd:YAG) laser for ultrasonic wave generation and a laser Doppler vibrometer for ultrasonic wave detection; (3) a state-space-based technique is introduced to reconstruct the state space attractor in the time domain under laser pulse excitation to localize the crack; and (4) through laser scanning, the crack is detected and visualized using the state-space-based technique.

This paper is organized as follows. In Sec. II, the working principle of nonlinear wave modulation due to pulse excitation is briefly reviewed followed by the description of the proposed damage-sensitive feature, Bhattacharyya distance (BD). Then, the experimental tests are described in Sec. III, and the proposed damage detection technique is applied to localization and visualization of actual fatigue cracks in aluminum plates in Sec. IV. Finally, the conclusion is provided in Sec. V.

II. THEORETICAL BACKGROUND

A. Nonlinear wave modulation using a single pulse excitation

It is known that, when two waves at distinctive frequencies $\omega_a$ and $\omega_b \ (\omega_a < \omega_b)$ propagate through a nonlinear region of a plate-like structure in the z-direction, the solution for the total particle displacement $u^T$ can be written as the summation of the linear response, harmonics and modulations [Fig. 1(a)] as follows:

$$
\begin{align*}
    u^T &= u_0 e^{i(k_a z - \omega_a t + \theta_a)} + u_0 e^{i(k_b z - \omega_b t + \theta_b)} + u_{2a} e^{2i(k_a z - \omega_a t + \theta_a)} \\
    &+ u_{2b} e^{2i(k_b z - \omega_b t + \theta_b)} + u_{2a2b} e^{2i(k_a z - \omega_a t + \theta_a)} e^{2i(k_b z - \omega_b t + \theta_b)} + \text{c.c.},
\end{align*}
$$

(1)

where c.c. stands for complex conjugate, $u_0$ and $u_b$ are the amplitudes of the linear waves at $\omega_a$ and $\omega_b$, $u_{2a}$ and $u_{2b}$ are the amplitudes of the nonlinear harmonics at $2\omega_a$ and $2\omega_b$, and $u_{2a2b}$ is the amplitude of the modulation responses at $\omega_b \pm \omega_a$, $\kappa_a$, $\kappa_b$ and $\theta_a$, $\theta_b$ are the wavenumbers and phases corresponding to waves a and b, respectively. For simplicity, the higher-order harmonics and modulations are omitted.

In general, two binding conditions must be satisfied for creation of modulated waves: 15,16 (1) synchronism condition—both the phase and group velocities of the linear waves must match with those of the modulated waves; and (2) non-zero power flux condition—the mode shapes of the linear waves should be matched with those of the modulated waves, ensuring non-zero power transfer from the linear waves to the modulated waves. These conditions can also be applied to nonlinear harmonics when $\omega_a = \omega_b$ and $\kappa_a = \kappa_b$.

In practice, it is challenging to find the optimal combination of two input frequencies, which satisfies all the binding conditions. In order to tackle this issue, a broadband excitation is used instead of two distinct input frequencies as an input signal. 11,18 When a broadband pulse signal is used as the input signal, nonlinear wave modulation can occur among various frequency components of the input signal and multiple frequency peaks are generated as shown in Fig. 1(b). Here, some of the frequency peaks in Fig. 1(b) could be the results of higher-order nonlinear modulations (cascade cross modulations) at the presence of cracks.19 However, it is difficult to relate the appearance of these additional peaks to the presence of a fatigue crack in an objective manner.

To solve this problem, a sideband peak count (SPC) technique is proposed to keep track of the relatively weak peaks in the neighborhood of the strong peaks generated by...
the material nonlinearity and/or the anomalies in the material. A feature called maximum SPC difference (MSPCD) is extracted from the frequency domain response to identify the existence of fatigue cracks. In this study, a crack detection technique is developed based on a state space attractor in the time domain to improve the probability of detection.

B. State space attractor

Assume a dynamical system described by a first order differential equation:

$$\dot{x} = f(x, t).$$

(2)

The solution to this problem with an initial value $x(0)$ will trace out a trajectory in state space defined by the system variable $x$. As transients die out, the trajectory will approach the system dynamical attractor, which is a geometric object in a state space to which all trajectories belong. The state space attractor is the geometric representation of the dynamical system, and thus will reflect a loss of dynamical similarity due to damage, especially the nonlinear one. The state space attractor will change if there is any variations in the dynamical system.

In practice, it is difficult to gain access to each of the system variables. Instead, based on the mathematical embedding proposed by Takens, the dynamics of the unobserved variables can be qualitatively reconstructed from a single time series of system response data. The reconstruction procedure is accomplished by concatenating lag copies of the single measured time series $x(n)$ ($n$ is the discrete time index, $n = 1, 2, ..., N$). In this study, $x(n)$ is a time series response corresponding to a broadband pulse laser input. Each discrete time instance of the attractor $X$ at time $n$ can be expressed as

$$X(n) = [x(n), x(n + T), ..., x(n + (m - 1)T)],$$

(3)

where $T$ is the time lag and $m$ is the embedding dimension. With proper time lag $T$ and embedding dimension $m$, a state space attractor, which preserves the “true” underlying system dynamics, can be reconstructed. For the selection of time lag $T$, there are two common techniques: one uses an autocorrelation function and the other uses the average mutual information (AMI) function. As for the selection of embedding dimension $m$, two commonly used methods are the singular system analysis and the false nearest neighbors (FNNs) method. Details on the aforementioned methods can be found in Overbey et al.

Recently, the geometric variations of data-driven dynamic state space attractors under deterministic or stochastic excitations have been studied for nonlinear damage detection by Nichols, Liu et al., Overbey et al., and Moniz et al. An attractor is first reconstructed in the state space from a scalar vibration response. Then, damage features representing geometrical changes of the attractor, such as prediction error, average local attractor variance ratio, correlation dimension of attractors, are extracted by comparing the attractors from the intact and damage conditions.

This study also uses the geometric variation of the reconstructed state space attractors to localize fatigue cracks. The major departure of this study from the aforementioned studies is that the state space attractor is applied to ultrasonic signals, the frequency band of which is far above the frequency ranges explored by the previous studies.

C. BD

In order to quantitatively indicate the geometric variation of the state space attractors, a statistical distance called BD is introduced as the damage-sensitive feature in this study.

The schematic overview of feature extraction from the reconstructed state space attractor is provided in Fig. 2. First, a set of fiducial points $y(i)$ ($i = 1, 2, ..., P$) is randomly selected from baseline attractor $Y$. The number of the fiducial points ($Q = N/100$ in this study) should be chosen so that the feature extraction results are insensitive to the addition of successive fiducial points. Next, a set of $P$ nearest neighbors $x_j(i)$ ($j = 1, 2, ..., P$) to each fiducial point $y(i)$ is found in the current attractor $X$. Here, $P$ should be selected so that the local dynamics of the attractor can be captured and it is also large enough to be insensitive to noise. In this study, $P$ is set to be $P = N/1000$. To prevent temporal correlations between the fiducial point and the selected neighbors, the Theiler window with step $2T$ is adopted so that any selected neighbor point will be separated from the fiducial point by at least $T$ points in time. Then, the fiducial points and neighbors are evolved in time with a time step $L$ ($L = 1$ in this study), and the mass centroid of the time-evolved neighborhood in the current attractor is computed by

$$\dot{y}(i + L) = \frac{1}{P} \sum_{j=1}^{P} x_j(i + L).$$

(4)

FIG. 2. (Color online) Schematic of feature extraction from the reconstructed state space attractor.
The error for each chosen fiducial point becomes

\[ e_i = \| y(i + L) - y(i + L) \|, \]

where \( \| \cdot \| \) denotes the Euclidean norm, and the total number of \( e_i \) is \( Q \). Finally, a statistical distance called BD is used to determine the difference of \( e \) obtained from the baseline and current conditions. BD is defined as

\[ \text{BD} = \frac{1}{Q} \left( \frac{(\mu_c - \mu_b)^2}{\sigma_c^2 + \sigma_b^2} + \frac{1}{2} \ln \left( \frac{\sigma_c^2 + \sigma_b^2}{2\sigma_c \sigma_b} \right) \right), \]

where \( \mu \) and \( \sigma \) represent the mean and standard deviation of \( e_i (i = 1, 2, \ldots, Q) \), and the subscripts \( b \) and \( c \) denote the baseline and current conditions, respectively. \( \mu_b \) and \( \sigma_b \) are calculated using the baseline attractor itself as the current attractor. This BD value is selected as the damage-sensitive feature in this study.

### III. EXPERIMENTAL INVESTIGATION

#### A. Specimen and fatigue testing

Two identical 3 mm thick aluminum plate specimens were fabricated using 6061-T6 aluminum alloy, and a notch was introduced on one edge of the specimen in the middle as shown in Fig. 3.

A fatigue crack was introduced to each specimen using an INSTRON 8801 fatigue testing system, as shown in Fig. 4(a). The specimens were tested under tension-tension cycling of a maximum load of 25 kN and a minimum load of 2.5 kN at a frequency of 10 Hz. The 15 mm long cracks were produced to two specimens after 18 793 and 20 209 loading cycles, respectively. The width of the fatigue cracks is less than 10 \( \mu \)m and even less than 5 \( \mu \)m near the crack tips, as shown in Fig. 4(b) and Fig. 4(c). These cracks are hardly detectable by traditional linear ultrasonic techniques.

#### B. Noncontact laser ultrasonic measurement system

A complete noncontact laser ultrasonic system is used in this study for generation and sensing of ultrasonic waves.\(^25\) As shown in Fig. 5, the excitation unit comprises a Q-switched Nd:YAG pulse laser, a galvanometer and a focal lens. The Nd:YAG laser employed in this study has a wavelength of 1064 nm and a maximum peak power of 3.7 MW, and generates a pulse input with 8 ns pulse duration at a repetition rate of 20 Hz. Note that, although the available maximum peak power is 3.7 MW, a peak power of around 0.2 MW is used for the experiment in this study. Ultrasonic waves are created through the thermal expansion of an infinitesimal area heated by the high power laser. Through the

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**FIG. 3.** (Color online) Dimensions of the specimen, crack location, and laser excitation and sensing arrangement: (a) six excitation-sensing paths, and (b) fixed point excitation and area scanning.

**FIG. 4.** (Color online) Fatigue test and microscopic images of aluminum specimens with fatigue cracks: (a) fatigue test setup, (b) microscopic image of crack tip in specimen I, and (c) microscopic image of crack tip in specimen II.
galvanometer, the pulse laser can be shot at the desired target excitation points on the specimen.

For the sensing unit, a commercial scanning LDV (Polytec PSV-400-M4) with a built-in galvanometer and an auto-focal lens is used. The laser source used for this LDV is a helium neon (He-Ne) laser with a wavelength of 633 nm. This one-dimensional (1D) LDV measures the out-of-plane velocity in the range of 0.01 um/s to 10 m/s over a target surface based on the Doppler frequency-shift effect of light.26 In this experiment, each ultrasonic velocity response is measured with a sampling frequency of 2.56 MHz for 25.6 ms, achieving a frequency resolution close to 40 Hz. Note that the LDV can also be used to acquire displacement, and the state space attractor can be reconstructed using displacement instead of velocity. However, the use of displacement is not explored in this study.

The control unit consists of a personal computer (PC), controller, velocity decoder with a maximum velocity sensitivity of 1 mm/s/V and a 14-bit digitizer with a maximum sampling frequency of 5.12 MHz. The controller sends out trigger signals to launch the excitation laser beam and to simultaneously start the data collection. In addition, the controller generates control signals to aim the excitation and sensing laser beams to the desired target positions.

Figure 6 shows the actual hardware components used in this experiment. The distances between the Nd:YAG laser head and the target specimen and between LDV and the target specimen are set to 1 m, which is the focal length of the used optical lens. To improve the signal to noise ratio, the responses are measured 100 times and averaged in the time domain.

C. Experimental setup

Two different experiments are conducted in this study. First, as shown in Fig. 3(a), six pairs of excitation and sensing laser beam points are selected to examine the sensitivity of the proposed damage feature, BD, extracted from each path to the fatigue crack. For the intact condition of each specimen, ultrasonic responses were recorded three times from each path. One of them was used as the reference signal, and the other two as the test signals acquired from the intact case. To take into account variations caused by resetting of the measurement system and the specimen after fatigue test, the whole measurement system was reconfigured even for the intact case. After the crack formation, ultrasonic time signals were collected again following the same measurement procedure as the intact case.

Second, in order to visualize the fatigue crack, the excitation point is fixed at a single point and a 3.5 cm × 3.5 cm area (with 361 scanning points) is scanned using LDV for ultrasonic sensing as shown in Fig. 3(b). Note that, only a small area covering the crack location is scanned in this study to readily visualize the nonlinearity near the crack. However, the whole target specimen should be scanned, because the crack location is not known in advance. The scanning is conducted before and after introducing the fatigue crack in the two specimens.

IV. EXPERIMENTAL RESULTS

For comparison with the proposed nonlinear feature, a linear feature called correlation coefficient is also calculated for the six paths [Fig. 3(a)]. Then, the crack visualization...
results from the scanned area [Fig. 3(b)] are presented using BD.

A. Correlation coefficient

Figure 7 displays a representative ultrasonic response obtained from path 1 of specimen I after 100 times averaging in both the time and frequency domains. The frequency content of the response spans up to 400 kHz. For comparison, the correlation coefficient of the two intact and one damage cases are computed with respect to the reference case. Figure 8 summarizes the correlation coefficient values calculated from six paths of the two specimens. It is concluded that the linear correlation coefficient is not an effective feature for fatigue crack detection because the difference between the damage and intact cases is small.

B. BD

Average mutual information (AMI) function and false nearest neighbors (FNNs) function are used to choose the optimal time lag $T$ and dimension $m$ for state space attractor reconstruction, respectively. For a given measured time series $x(n) (n = 1, 2, \ldots, N)$, the AMI function measures the average amount of information learned about $x(n + T)$ by measuring $x(n)$. If $x(n + T)$ is completely independent from $x(n)$, AMI value becomes zero. Figure 9(a) shows the AMI values of the reference responses obtained from six paths of specimen I with response to time lag $T$. For all six paths, the AMI values approach to almost zero when the time lag $T$ is near 13 – 17. Therefore, $T = 15$ is selected in this study.

If the distance between two closely positioned points substantially increase as the embedded dimension increases from $d$ to $d+1$ dimensional space, this pair of points is named a false neighbor due to improper unfolding. As the embedded dimension expands, the percentage of false nearest neighbors declines to a suitably small value. The lowest embedded dimension, which corresponds to the first near zero percentage of FNNs, is often chosen as the appropriate embedding dimension. The FNNs functions for the reference responses of six paths in specimen I are depicted in Fig. 9(b). The percentage of FNNs functions decline rapidly and approaches to the minimum values at $m = 5$ for all six paths. Therefore, the embedded dimension is selected as 5 or higher. Note that the FNNs value does not converge to zero even after $m$ increases over 5. Because the laser pulse excitation used in this study is neither a purely stochastic process nor a fully deterministic sequence, there are still some information retained in higher dimensions although the majority of information is contained in the first few dimensions.

First, the effect of the embedded dimension on crack detection is investigated. The state space attractors for the damage and intact cases are reconstructed with varying dimensions ($m = 1, 2, 3, 4, 5, 10, 15,$ and 20) and compared with the baseline attractors with the same dimension for all paths in specimen I. Then, the BD is calculated to assess the deviation of the current (either intact or damage) attractor from the baseline one in a statistical sense. The results are summarized in Fig. 10. As the dimension increases beyond 5, the damage and intact attractors are clearly distinguished and the wave propagation path (path 2) crossing the crack tip is easily identified. The dimension $m$ of 20 is selected for subsequent analyses.

Figure 11 shows the BD values of the state space attractors reconstructed from specimens I and II with $m = 20$. In all cases, the location of the crack tip is clearly identified by a sudden increase of the BD values. For comparison, the maximum sideband peak count difference (MSPCD) is also calculated and shown in Fig. 12. The comparison of Figs. 11 and 12 reveals that the BD outperforms the

FIG. 8. (Color online) Damage localization using correlation coefficient obtained from (a) specimen I and (b) specimen II.

FIG. 9. (Color online) Parameter selection for state space attractor reconstruction: (a) average mutual information (AMI) for time lag $T$ and (b) false nearest neighbors (FNNs) for dimension $m$. 

FIG. 10. (Color online) The effect of embedded dimension $m$ on Bhattacharyya distance (BD) and fatigue crack localization ($m = 1, 2, 3, 4, 5, 10, 15, 20$, obtained from specimen I).

FIG. 11. (Color online) Bhattacharyya distance (BD) values obtained from (a) specimen I and (b) specimen II.

FIG. 12. (Color online) Maximum sideband peak count difference (MSPCD) values obtained from (a) specimen I and (b) specimen II.
MSPCD. However, the calculation of BD is computationally more demanding than the calculation of MSPCD.

Note that, MSPCD value significantly increased even when the path did not pass through the crack in Liu et al.18 because the power level of the laser excitation was much higher (0.4 MW peak power). On the other hand, the power level of the excitation laser beam is kept below 0.2 MW in this study so that the laser excitation can “activate” crack opening and closing only when the propagating waves directly pass through the crack.

C. Fatigue crack visualization using BD

As stated above, an area scanning test is also conducted in specimen I and II before and after introducing the fatigue crack [Fig. 3(b)]. Totally, responses from 361 (19 x 19) sensing points are acquired by the noncontact laser ultrasonic measurement system. The scanning takes around 30 min (361 points x 100 averaging x 1/20 s for each scanning point, using a laser with 20 Hz repetition rate). BD values for all sensing points are calculated. The total computation takes a longer time as the embedded dimension m increases (15 and 60 min for m = 5 and 20, respectively, using 3.6 GHz CPU and 4 GB RAM). Then, the crack visualization result is acquired using the BD values (m = 20) and presented in Fig. 13. It is clearly demonstrated that strongest nonlinearity is observed near the crack tip.27 The visualization results further imply that the proposed technique can not only detect the crack but also localize it. Although it is not demonstrated in this study, the proposed technique is expected to detect even subsurface crack if the energy of the excitation laser beam is high enough to “activate” the opening and closing of the hidden crack.

V. CONCLUSIONS

This study demonstrates that laser nonlinear wave modulation spectroscopy (LNWMS) can be successfully applied for fatigue crack localization in metallic structures. Different from the conventional nonlinear wave modulation techniques, the proposed LNWMS technique takes advantage of a single pulse excitation and a complete noncontact laser ultrasonic system is used for LNWMS measurement. A new feature acquired from the corresponded ultrasonic responses in the time domain is proposed for detecting the nonlinearity caused by a fatigue crack and localizing it. This feature called BD is obtained by reconstructing state space attractors and used for localization and visualization of the fatigue crack. However, there are still a number of challenges associated with laser ultrasonic techniques such as eye-safety, material ablation, and surface treatment. The proposed LNWMS will be further advanced for reference-free damage detection and visualization by comparing the state space attractor at a specific spatial point with adjacent spatial points rather than with its pristine condition. In this way, the effects of varying operational and environmental variations in field applications can be minimized. Also, a future study is warranted to quantify the size of the crack and remaining useful life of the specimen.

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