Development of a mixed pixel filter for improved dimension estimation using AMCW laser scanner

Qian Wang a,b, Hoon Sohn b,⇑, Jack C.P. Cheng a

a Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong
b Department of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology, South Korea

Abstract

Accurate dimension estimation is desired in many fields, but the traditional dimension estimation methods are time-consuming and labor-intensive. In the recent decades, 3D laser scanners have become popular for dimension estimation due to their high measurement speed and accuracy. Nonetheless, scan data obtained by amplitude-modulated continuous-wave (AMCW) laser scanners suffer from erroneous data called mixed pixels, which can influence the accuracy of dimension estimation. This study develops a mixed pixel filter for improved dimension estimation using AMCW laser scanners. The distance measurement of mixed pixels is firstly formulated based on the working principle of laser scanners. Then, a mixed pixel filter that can minimize the classification errors between valid points and mixed pixels is developed. Validation experiments were conducted to verify the formulation of the distance measurement of mixed pixels and to examine the performance of the proposed mixed pixel filter. Experimental results show that, for a specimen with dimensions of 840 mm x 300 mm, the overall errors of the dimensions estimated after applying the proposed filter are 1.9 mm and 1.0 mm for two different scanning resolutions, respectively. These errors are much smaller than the errors (4.8 mm and 3.5 mm) obtained by the scanner’s built-in filter.

Keywords:
Mixed pixel filter
AMCW laser scanner
Phase shift measurement
Dimension estimation

1. Introduction

Accurate dimension estimation is desired in many fields, such as manufacturing, surveying and civil engineering. Traditionally, dimension estimation is performed by manual inspection with devices, such as measuring tapes. However, such manual inspection is time-consuming and labor-intensive (Phares et al., 2004). In recent years, electronic instruments, including laser range finders and total stations, have been widely used. Although these instruments are more convenient and efficient, they can acquire measurement data only from a few specified points, one by one. Thus, it is still time-consuming to obtain measurement data from a large area with high resolution.

In the recent decades, 3D laser scanners have gained popularity because they can automatically acquire range measurement data at a high speed and with high accuracy. Various applications of laser scanners have been reported, including reverse engineering (Chang and Chang, 2002; Son et al., 2002), 3D model reconstruction (Tang et al., 2010; Bosch and Haas, 2008), construction progress tracking (Turkan et al., 2012; El-Omari and Moselhi, 2008), and dimension estimation (Bosche, 2010; Wang et al., 2016). Among laser scanners with different range measurement principles, time-of-flight (TOF) and amplitude-modulated continuous-wave (AMCW) scanners are most popular (Amann et al., 2001; Schulz, 2007). Other types of laser scanners, such as triangulation based scanners and frequency-modulated continuous-wave (FMCW) scanners, are not discussed here because of their limited applications. TOF scanners measure the distance to a target by emitting a laser pulse and detecting the returned pulse, and the distance is computed based on the arrival time of the returned pulse. AMCW scanners emit amplitude-modulated, continuous waves and measure the phase difference, which is known as the phase shift, between the emitted and the returned signals. The distance is then obtained based on the phase shift and the wavelength of the continuous wave. Typically, TOF scanners have longer scanning distances (more than 1 km) than AMCW scanners (approximately 100 m), whereas AMCW scanners have higher range measurement accuracy (a few millimeters) than TOF scanners (a few centimeters). Therefore, AMCW scanners are more suitable for dimension estimation at short ranges.

Errors associated with AMCW scanners are investigated by several researchers. Lichti (2007) and Reshetyuk (2010) formulated...
models describing systematic errors in AMCW scanners and developed self-calibration methods to reduce these errors considering correlations among scanner parameters. Soudarissanane et al. (2011) investigated the effects of scanning geometry, such as incident angle and the distance from a scanner to an object, on measurement accuracy and signal to noise ratio. Hebert and Krotkov (1992) investigated the effect of mixed pixels and applied a median filter to remove the mixed pixels. A mixed pixel occurs when a laser beam falls at the edge of an object. Then, the laser beam is split into two parts, and the split beams fall on two different surfaces, namely, the foreground surface and the background surface. The returned signals from the two surfaces are both received by the scanner, and the resulting scan point becomes a mixed pixel. The location of a mixed pixel can be anywhere along the direction of the laser beam. As shown in Fig. 1, type 1 mixed pixels are located between the foreground and the background surfaces; type 2 are located in front of the foreground surface; and type 3 are located behind the background surface. Because the existence of mixed pixels can influence the accuracy of dimension estimation, it is necessary to remove mixed pixels before estimating dimensions.

Several algorithms have been developed to remove mixed pixels from laser scan data. In an early investigation, Hebert and Krotkov (1992) suggested using median filters or removing isolated points because most mixed pixels have range measurements far from the true value. However, the two approaches cannot remove mixed pixels that are close to the actual edges of the foreground surface. Adams and Probert (1996) developed a physical model for the mixed pixels problem and proposed an algorithm to detect mixed pixels. However, their algorithm is not universally applicable because it requires that the footprints of two adjacent scan points overlap. In addition, the quantitative performance of the algorithm was not provided. Tuley et al. (2005) developed an algorithm capable of removing mixed pixels from scan data of terrain with vegetation. However, the developed algorithm is applicable only to thin structures, such as pipes. Tang et al. (2007) quantitatively compared five different algorithms for mixed pixels detection. The results showed that there is no best algorithm, and all of the algorithms have difficulty in detecting mixed pixels that are close to the actual edges. Furthermore, the performance of all five algorithms relies on setting proper thresholds, but no instruction is provided on how to determine the optimal thresholds.

To improve the accuracy of dimension estimation using AMCW laser scan data, this study proposes a mixed pixel filter based on the formulation of the distance measurement of mixed pixels. The developed filter can minimize the classification errors when scan data is classified into mixed pixels and valid points. The uniqueness of this study includes (1) the distance measurement of mixed pixels using multiple modulated waves is formulated and quantitatively verified by experiments, and (2) a mixed pixel filter is proposed based on the formulation of distance measurement and is quantitatively evaluated by experiments. The rest of this paper is organized as follows. Section 2 formulates the distance measurement of mixed pixels based on the working principle of AMCW laser scanners. Section 3 develops a mixed pixel filter based on the formulation illustrated in Section 2. In Section 4, validation experiments were conducted to verify the formulation of the distance measurement of mixed pixels and to examine the performance of the developed mixed pixel filter. Lastly, Section 5 summarizes this study and suggests future work.

2. Formulation of the distance measurement of mixed pixels

2.1. Working principle of AMCW laser scanners

AMCW laser scanners measure distances using the phase-shift method:

\[ D = \frac{\lambda}{4\pi} \Delta \phi \]  \hspace{1cm} (1)

where \( D \) is the distance from the scanner to a target, \( \lambda \) is the wavelength of the amplitude-modulated wave, and \( \Delta \phi \) is the phase shift between the emitted and returned signals (Hebert and Krotkov, 1992).

Because \( \Delta \phi \) is defined in \([0, 2\pi]\), corresponding to the maximum value \( 2\pi \), the scanner can correctly measure distances, i.e., yielding unambiguous measurements, only within an ambiguity interval \( D_a \):

\[ D_a = \frac{\lambda}{2} \]  \hspace{1cm} (2)

When the actual distance is over the ambiguity interval, the distance measurement is incorrect, yielding an ambiguous measurement.

Generally, when using a modulated wave with wavelength \( \lambda \) to measure distance \( D \), the measured phase shift \( \Delta \phi(\lambda) \) and the measured distance \( X(\lambda) \) are related to \( D \) as follow:

\[ \Delta \phi(\lambda) = \frac{4\pi}{\lambda} D - 2n\pi \]  \hspace{1cm} (3)

\[ X(\lambda) = D - \frac{n\lambda}{2} \]  \hspace{1cm} (4)

where \( n \) is an integer such that \( 0 \leq \Delta \phi(\lambda) < 2\pi \) or \( 0 \leq X(\lambda) < \lambda/2 \).

One approach to increase the ambiguity interval is to use a modulated wave with a longer wavelength. However, because the measured phase shift \( \Delta \phi \) has a certain measurement precision, the precision of the distance measurement deteriorates as the wavelength increases, according to Eq. (1). To achieve a large ambiguity interval and a high precision measurement at the same time, AMCW laser scanners often use modulated waves with two or more wavelengths (FARO, 2016). The longer wave is used to correct the distance measurement of the next shorter wave in case the shorter wave gives an ambiguous measurement. In this way, the total ambiguity interval of the scanner is determined by the longest wave, and the precision of the distance measurement is determined by the shortest wave.

Assuming that a scanner emits \( N \) modulated waves with decreasing wavelengths of \( \lambda_1, \lambda_2, \ldots \) and \( \lambda_N \), the total ambiguity interval is \( \lambda_1/2 \), determined by the longest wave. Each wave with wavelength \( \lambda_i \) \((i = 1, 2, \ldots N)\) gives a distance measurement \( X(\lambda_i) \) according to Eq. (4). For the longest wavelength \( \lambda_1 \), the measurement \( X(\lambda_1) \) is not ambiguous as long as the actual distance falls within the total ambiguity interval. For wavelength \( \lambda_2 \), an integer, \( a_1 \), is selected and added to \( X(\lambda_2) \) such that \( X(\lambda_2) + a_1\lambda_2/2 \) becomes the closest to \( X(\lambda_1) \). In this way, \( X(\lambda_2) \) is corrected to \( X(\lambda_2) + a_1\lambda_2/2 \), and the updated \( X(\lambda_2) \) value is used to correct \( X(\lambda_3) \). This correction...
is recursively repeated in the order of $\lambda_2, \lambda_3, \ldots, \lambda_n$. For the shortest wavelength $\lambda_n$, the corrected measurement, $X(\lambda_n) + \alpha_0, 1/\lambda_n^2/2$, has the highest precision and is taken as the final distance measurement $X_f$. The above correction process for obtaining the final distance measurement $X_f$ is defined using a function, $C$:

$$X_f = C(X(\lambda_1), X(\lambda_2), \ldots, X(\lambda_n))$$  \hspace{1cm} (5)

Fig. 2 illustrates the distance measurement using multiple modulated waves. The scanner uses three modulated waves with wavelengths of 50 m, 10 m, and 1 m. The actual distance from the scanner to the target is 3.412 m. The distances measured by the three waves, denoted as $X(50)$, $X(10)$, and $X(1)$, are 3.40 m, 3.41 m, and 0.412 m, respectively. It is assumed that the precision of a measurement $X(\lambda_i)$ is proportional to the wavelength $\lambda$, i.e., precision $= 0.001 \lambda_i$. First, $X(50)$ provides an unambiguous distance measurement of 3.40 m. Next, $X(10)$ provides a distance measurement of 3.41 m (3.41 + 5.5a1, $a_1 = 0$). Lastly, $X(1)$ gives the final distance measurement of 3.412 m (0.412 + 0.5a2, $a_2 = 6$).

2.2. Distance measurement of a mixed pixel

Fig. 3 illustrates the appearance of a mixed pixel when a laser beam $(\text{Fig. 3(a)})$ emitted at the edge of the foreground surface is split into two parts so that one part $(A_1$ in Fig. 3(b)) falls on the foreground surface and the other part $(A_2$ in Fig. 3(b)) falls on the background surface. The distances from the scanner to the two surfaces are denoted as $D_1$ and $D_2$, respectively. The difference between $D_1$ and $D_2$ is denoted as $\Delta D$, and the distance measurement of the mixed pixel is denoted as $D_m$.

2.2.1. Distance measurement using one modulated wave

The distance measurement of a mixed pixel has been formulated considering one modulated wave (Adams and Probert, 1996; Hancock, 1999). The emitted laser intensity $z_0$ is assumed to be a cosine function:

$$z_0 = I_0 \cos \left(\frac{2\pi c}{\lambda_0} t\right)$$  \hspace{1cm} (6)

where $I_0$, $c$, $\lambda_0$ and $t$ denote the intensity, speed and wavelength of the emitted wave, and time, respectively. Here, $D_1$ and $D_2$ are assumed to be within the ambiguity interval.

The returned signals from the foreground and the background surfaces are denoted as $z_1$ and $z_2$, respectively:

$$z_1 = I_1 \cos \left(\frac{2\pi c}{\lambda_1} t + \phi_1\right), \quad i = 1, 2$$  \hspace{1cm} (7)

where $I_1$ and $\phi_1$ are the intensity and phase of $z_0$, respectively.

Because the phase of $z_0$ is 0, the measured phase shift of $z_1$ is $\phi_1$. According to Eq. (3), $\phi_1$ is related to $D_1$ as follows:

$$\phi_1 = \frac{4\pi}{\lambda_0} D_1, \quad i = 1, 2$$  \hspace{1cm} (8)

Because the scanner receives both $z_1$ and $z_2$, the resulting signal $z$ becomes:

$$z = z_1 + z_2$$  \hspace{1cm} (9)

When $z$ is represented as a cosine function:

$$z = I_1 \cos \left(\frac{2\pi c}{\lambda_1} t + \phi\right)$$  \hspace{1cm} (10)

where $I_1$ and $\phi$ are the intensity and phase of $z$, respectively. The phase $\phi$ can be obtained from Eqs. (7), (9), and (10) as:

$$\cos \phi = -\frac{(I_1 \sin \phi_1 + I_2 \cos \phi_2)}{\sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos(\phi_1 - \phi_2)}}$$  \hspace{1cm} (11)

Finally, the distance measurement $D_m$ of the mixed pixel is solved from Eq. (1) as:

$$D_m = \frac{\lambda_0}{4\pi} \phi$$  \hspace{1cm} (13)

The summation of two returned signals can also be explained by geometric interpretation (Hancock, 1999). As shown in Fig. 4, $z_1$ and $z_2$ are represented by two vectors. The length of a vector corresponds to the intensity of the signal, and the angle of a vector corresponds to the phase of the signal. Then, $z$ becomes the sum of the two vectors, and its phase $\phi$ is obtained from its angle. As shown in Fig. 4(a), when the difference between $\phi_1$ and $\phi_2$ is less than $\pi$, the phase $\phi$ falls between $\phi_1$ and $\phi_2$. According to Eqs. (8) and (13), the mixed pixel is located between the foreground and the background surfaces ($D_1 < D_m < D_2$). When the difference between $\phi_1$ and $\phi_2$ is larger than $\pi$, the resulting signal is such that either $\phi < \phi_1 < \phi_2$ (Fig. 4(b)) or $\phi_1 < \phi_2 < \phi$ (Fig. 4(c)), that is, the mixed pixel is either in front of the foreground surface ($D_m < D_1 < D_2$) or behind the background surface ($D_1 < D_2 < D_m$).

2.2.2. Distance measurement using multiple modulated waves

For a scanner emitting N modulated waves with decreasing wavelengths of $\lambda_1, \lambda_2, \ldots, \lambda_n$, there will be a total of 2N returned signals from the two surfaces. Firstly, for each wavelength $\lambda_i (1 = 1, 2, \ldots, N)$, the two returned signals are mixed to generate a distance measurement $X(\lambda_i)$, as illustrated in Section 2.2.1. Secondly, all N distance measurements from different waves are processed by the correction function $C$, as described in Eq. (5), to generate the final distance measurement.

Fig. 5 illustrates the distance measurement of a mixed pixel, considering two waves with wavelengths of 50 m and 10 m. The distances $D_1$ and $D_2$ are assumed to be 2 m and 6 m, respectively. For the 50 m wave, according to Eqs. (3) and (4), the phases of the two returned signals are $4\pi/25$ and $12\pi/25$, respectively, which correspond to distance measurements of 2 m and 6 m, respectively. Therefore, the phase of the resulting signal falls between $4\pi/25$ and $12\pi/25$, and the distance measurement $X(50)$ falls between 2 m and 6 m. Similarly, for the 10 m wave, the phase of the resulting signal falls between $2\pi/5$ and $4\pi/5$, and the distance measurement $X(10)$ falls between 1 m and 2 m. According to Eq. (5), the final distance measurement is either $X(10)$ or $5 m + X(10)$, depending on the specific $X(50)$ and $X(10)$ values. In the former case, the mixed pixel is in front of the background surface, as $X(10) < 2 m$. In the latter case, the mixed pixel is behind the background surface, as $5 m + X(10) > 6 m$.

As stated above, the distance measurement $D_m$ of a mixed pixel is determined by three factors: (1) the wavelength $\lambda_i$ of the modulated wave, (2) the phases, $\phi_i$ and $\phi_2$, of the returned signals, which are determined by $\lambda_i$, $D_1$, and $D_2$, and (3) the intensities, $I_1$ and $I_2$, of the returned signals, more specifically, the ratio of $I_1/I_2$ according to Eqs. (11) and (12). To conclude, $D_m$ is a function of $\lambda_i$, $D_1$, $D_2$, and $I_1/I_2$ defined as $H$:

$$D_m = H(\lambda_i, D_1, D_2, I_1/I_2)$$  \hspace{1cm} (14)

Once the scanning parameters are established, the $\lambda_i$, $D_1$, and $D_2$ values are fixed. The only unknown becomes $I_1/I_2$, and the distance measurement is determined by four factors: (1) the reflectance values of the two surfaces, (2) the incident angles of the emitted laser beams with respect to the two surfaces, (3) the distances to the two surfaces, and (4) the emitted intensities of the two parts of the laser beam that fall on the two surfaces (Nitzan et al., 1977). For each specific scan, all of the aforementioned factors, except the last, are constant. For the split laser beam shown in Fig. 3(b),
the last factor is related to the areas $A_1$ and $A_2$ of the two parts of the split laser beam. Furthermore, $A_1$ and $A_2$ are both related to $x$, which denotes the distance from the edge to the farthest endpoint of part $A_1$. Here, $x$ is defined within $[0, d]$, where $d$ is the size of the laser beam. Note that, once $x$ is known, $I_1/I_2$ and $D_m$ can be estimated. Therefore, $D_m$ can be regarded as a function of $x$.

2.2.3. Relation between $x$ and $D_m$

To further investigate the relation between $x$ and $D_m$, assumptions are made as follows: (1) The scanner uses three modulated waves with wavelengths of 307 m, 19.2 m and 2.4 m, which are determined according to the specific scanner, a FARO Focus 3D 120 scanner (FARO, 2015), used in this study. (2) This study focuses on short range scanning where $D_1$ and $D_2$ are less than 10 m and $D_m$ is less than 2.4 m. Based on these assumptions, the $x - D_m$ relation with different $D_1$ and $D_2$ values is theoretically predicted based on the formulation illustrated in Section 2.2.2. For simplicity, this sub-section further assumes that the ratio $I_1/I_2$ equals to the area ratio $A_1/A_2$. Note that this assumption can have an overall impact on the locations of mixed pixels. If the actual ratio $I_1/I_2$ is larger than $A_1/A_2$, mixed pixels can be overall closer to the foreground surface, because the foreground surface provides a stronger returned signal. By analyzing the $x - D_m$ relations, four conclusions are drawn, as follows.

(1) Four distinctive relations between $x$ and $D_m$ exist depending on which quarter of the shortest wavelength of 2.4 m the $\Delta D$ value falls in. Fig. 6(a)–(d) illustrates the $x - D_m$ relation in a
solid line for four different $\Delta D$ ranges. When $\Delta D < 0.6$ m (Fig. 6(a)), mixed pixels fall between the foreground and the background surfaces. When $0.6$ m $< \Delta D < 1.2$ m (Fig. 6(b)), mixed pixels are either in front of the foreground surface or behind the background surface. When $1.2$ m $< \Delta D < 1.8$ m (Fig. 6(c)), mixed pixels again fall between the two surfaces but are not continuously located therein. Instead, some mixed pixels are close to the foreground surface and the others are close to the background surface, showing a gap in $D_m$. When $1.8$ m $< \Delta D < 2.4$ m (Fig. 6(d)), mixed pixels are located within three narrow ranges, which are separated by two gaps. For the latter three cases ($0.6$ m $< \Delta D < 2.4$ m), the height of all the gaps in $D_m$ is $0.6$ m, which is a quarter of the shortest wavelength $2.4$ m.

(2) For all $\Delta D$ values, when the $x$ value becomes small, mixed pixels approach the background surface; when the $x$ value becomes large, mixed pixels approach the foreground surface.

(3) When $\Delta D > 0.6$ m, there are always some mixed pixels located particularly close to the foreground surface, which are marked as “mixed pixels of concern” in Fig. 6(b)-(d), and these mixed pixels are separated from the other mixed pixels by a gap in $D_m$. These pixels are of the most concern because they cannot be easily distinguished from the valid points. When $\Delta D < 0.6$ m, all of the mixed pixels are defined as “mixed pixels of concern”. For a specific $\Delta D$ value, the “mixed pixels of concern” are either all in front of the foreground surface or all behind the foreground surface.

(4) For any specific $\Delta D$ value, there is always a one-to-one match between $x$ and $D_m$.

3. Development of the mixed pixel filter

To remove mixed pixels from the scan data, a mixed pixel filter is developed based on the formulation of the distance measurement of mixed pixels. The scan data can be classified into three categories: (1) valid points that belong to the foreground surface, (2) mixed pixels, and (3) background points that belong to the background surface. The developed filter aims to remove the mixed pixels and background points while retaining the valid points.

It is assumed that the scan data are located in the coordinate system shown in Fig. 3(a), where the foreground surface is taken as the X–Y plane. The Z axis is perpendicular to the foreground surface and orients from the foreground towards the background surface. The developed filter operates on the $Z$ values of the scan data because the valid points, mixed pixels, and background points have different ranges of $Z$ values. $Z_v$ denotes the $Z$ value of a valid point, which is supposed to be 0 for a smooth foreground surface.

However, due to the surface roughness, the actual $Z_v$ value varies within a certain range. The $Z$ value of a mixed pixel, denoted as $Z_m$, is obtained from $D_m$:

$$Z_m = (D_m - D_1) \cos \alpha$$

where $\alpha$ is the incident angle of the laser beam with respect to the foreground surface, as shown in Fig. 3(a). The $Z$ value of a background point is denoted as $Z_b$, which equals to $(D_2 - D_1) \cos \alpha$ but may vary within a certain range due to the surface roughness.

A laser scanner scans an object by rotating horizontally and vertically. When the scanner rotates and scans across an edge, the number of mixed pixels is determined by the size $d$ of the laser beam and the spacing $s$ between adjacent scan points. According to Tang et al. (2009), $d$ is determined as:

$$d = (d_0 + (L - L_0) \alpha_0) / \cos \theta \cdot \cos \alpha$$

where $d_0$ is the size of the laser beam at the exit, $L$ is the scanning distance, $L_0$ is the distance to the focal point, $\alpha_0$ is the laser divergence, $\alpha$ is the rotation rate of the scanner, and $\theta$ is the sampling time of a scan point. Here, $L$ equals $D_1$ because the laser beam is split at the edge of the foreground surface. $\cos \theta \cdot \cos \alpha$ results from the rotation of the scanner. Because the scanner’s vertical rotation is much faster than horizontal rotation, the vertical size of the laser beam is larger than the horizontal size. The spacing $s$ between adjacent scan points is obtained as:

$$s = \theta L / \cos \alpha$$

where $\theta$ is the angular resolution of the scanner. Two different mixed pixel filtering algorithms for $s \geq d$ and $s < d$ are developed in Sections 3.1 and 3.2, respectively.

3.1. Case 1: Mixed pixel filter when $s \geq d$

When $s \geq d$, two adjacent scan points do not overlap so that there is at most one mixed pixel in a row of scan points. Fig. 7 shows an example where a scanner scans horizontally across a vertical edge, and points A and B represent two adjacent scan points. Assuming that point A is a valid point and point B is a non-valid point, which can be a mixed pixel or a background point, P1, P2, and P3 describe three distinctive positions of points A and B with respect to the edge. The scan points can be anywhere between P1 and P3 with equal probabilities, and point B becomes a mixed pixel only when points A and B fall between P1 and P2. When the scan points fall between P2 and P3, point B becomes a background point, and there is no mixed pixel in this row. Generalizing this observation, when $s \geq d$, the probability that there is a mixed pixel in a row is $d/s$, and the counterpart probability that there is
no mixed pixel in a row is \((s - d)/s\). In the former case, the \(x\) value of the mixed pixel can be equally anywhere between 0 and \(d\).

The developed mixed pixel filter for case 1 includes the following four steps.

**Step 1:** Estimate the probability distribution function (PDF) of \(Z_v\), denoted as \(f(Z_v)\). Some valid points are selected from the scan data and their \(Z\) values are fitted into a normal distribution, which is then taken as the estimation of \(f(Z_v)\).

**Step 2:** Estimate the PDF of \(Z_m\), denoted as \(f(Z_m)\). Because the \(x - D_m\) relation is obtained in Section 2 and the \(Z_m - D_m\) relation is known as Eq. (15), \(Z_m\) can be obtained as a function of \(x\), denoted as \(F(x)\). First, the cumulative probability function (CDF) of \(Z_m\), denoted as \(F_{Z_m}(Z)\), is solved as

\[
F_{Z_m}(Z) = 
\begin{cases} 
0 & \text{if } Z < Z_{m1} \\
1 & \text{if } Z > Z_{m2} 
\end{cases}
\]

where \(Z_{m1}\) and \(Z_{m2}\) are the \(Z\) values of the mixed pixels. The PDF of \(Z_m\) is obtained as the differential of \(F_{Z_m}(Z)\), i.e.,

\[
dF_{Z_m}(Z) = \frac{d}{dz} F_{Z_m}(Z).
\]

**Step 3:** Filter out obvious non-valid points, which include (1) mixed pixels far from the foreground surface and (2) background points, by removing scan points with \(Z\) values over a certain threshold \(T_1\). When \(\Delta D < 0.6\) m (Fig. 8(a)), \(T_1\) is set such that \(P(Z_m < T_1) = 99\%\) based on \(f(Z_m)\) obtained in step 1. When \(\Delta D > 0.6\) m, \(T_1\) is set to any value between the \(Z\) values of “mixed pixels of concern” and the \(Z\) values of the other mixed pixels, as shown in Fig. 8(b)–(d). Meanwhile, \(T_1\) value must ensure that valid points are not removed. Because the gap \((0.6 \cos \alpha m)\) in \(Z_m\) is at least one order of magnitude larger than the typical values of \(Z_v\) (a few centimeters at most), the desired \(T_1\) value can easily be found. After step 3, the \(Z\) values of the remaining mixed pixels are either all positive (Fig. 8(a) and (c)) or all negative (Fig. 8(b) and (d)). The probability that a mixed pixel is remained in the scan data is denoted as \(g\) and equals to the probability that \(Z_m < T_1\), i.e.,

\[
\frac{dG(Z_m)}{dz} = \frac{d}{dz} F_{Z_m}(Z) = \frac{d}{dz} \left(1 - \frac{d}{dz} F_{Z_v}(Z)\right).
\]

**Step 4:** Classify scan points as either valid points or mixed pixels based on another filtering threshold \(T_2\). Because there is at most one mixed pixel in each row, only the last scan point in each row needs to be classified. Here, there are two types of classification errors: (1) type I error, \(e_1\), where a valid point is classified as a mixed pixel, and (2) type II error, \(e_2\), where a mixed pixel is classified as a valid point. As shown in Fig. 9, two curves represent the PDF of the \(Z\) value of the last point in each row, when it is a valid point \((1 - \frac{d}{dz} F(Z_v))\) or a mixed pixel \((\frac{d}{dz} F(Z_m))\).
respectively. Note that \( f(Z_m) \) shown in Fig. 9 only considers the remaining mixed pixels from step 3 and the figure is renormalized. If the \( Z \) value of the last point in each row is larger than \( T_2 \), the point is classified as a mixed pixel. Otherwise, it is classified as a valid point. Therefore, the probability of \( e_1 \), denoted as \( P(e_1) \), equals to \( P(\text{being a valid point}) \times P(\text{a valid point being misclassified}) \), i.e., \( \left(1 - \frac{d}{s}\right) \times P(Z_v > T_2) \). The probability of \( e_2 \), denoted as \( P(e_2) \), equals to \( P(\text{being a mixed pixel}) \times P(\text{a mixed pixel being misclassified}) \), i.e., \( \frac{dn}{s} \times P(Z_m < T_2) \). As shown in Fig. 9, two shadow areas with different patterns equal to \( P(e_1) \) and \( P(e_2) \), respectively, and the total area equals to \( P(\text{error}) \). \( T_2 \) is selected so that \( P(\text{error}) \) is minimized and such \( T_2 \) value should be located at the crossing point of the two PDF curves.

\[
P(\text{error}) = P(e_1) + P(e_2) = \left(1 - \frac{d}{s}\right) \times P(Z_v > T_2) + \frac{dn}{s} \times P(Z_m < T_2)
\]

(18)

3.2. Case 2: Mixed pixel filter when \( s < d \)

When \( s < d \), two adjacent scan points overlap so that there is at least one mixed pixel in each row. When a scanner scans horizontally across a vertical edge, as shown in Fig. 10, points A, B, and C represent three consecutive scan points. It is assumed that point A is a valid point, whereas points B and C are the first and second non-valid points, respectively. Hence, the positions of the scan points with respect to the edge can be equally anywhere between P1 and P3. Between P1 and P3, point B is always a mixed pixel, and its \( x \) value can be anywhere within \([d - s, d]\) with equal probabilities. Point C becomes a mixed pixel between P1 and P2 with a probability of \((d - s)/s\) and becomes a background point between P2 and P3 with a probability of \((2s - d)/s\). When point C is a mixed pixel, its \( x \) value can be anywhere within \([0, d - s]\) with equal probabilities.

More generally, when \( d/s \) falls between two integers \( n_m - 1 \) and \( n_m \), there will be \( n_m \) non-valid points in each row. The 1st to the \((n_m - 1)\) non-valid points are always mixed pixels, and the \( n_m \) non-valid point can be either a mixed pixel or a background point. For the \( i \)th \((1 < i < n_m - 1)\) non-valid point, its \( x \) value can be equally anywhere within \([d - is, d - (i - 1)s]\). For the \( n_m \) non-valid point, the probability of being a mixed pixel is \((d - n_ms + s)/s\), and its \( x \) value can be equally anywhere within \([0, d - (n_m - 1)s]\). Then, the probability of the \( n_m \) non-valid point being a background point becomes \((n_ms - d)/s\).

As stated above, the mixed pixels in each sequence (e.g., 1st or 2nd) have an exclusive range of \( x \). Because the \( x - D_m \) relation is a one-to-one function for a specific scan and \( Z_m \) is linearly related to
D_m, the mixed pixels in each sequence have an exclusive range of Z_m. Furthermore, since the 1st mixed pixel in each row has the largest x value, it is more likely to be closest to the foreground surface, as concluded in Section 2.2.3. On the other hand, the 2nd and subsequent mixed pixels are more likely to be far from the foreground surface. Based on these observations, the mixed pixel filter for case 2 is developed as follows.

**Step 1:** Filter out all the non-valid points except the 1st mixed pixel in each row. Firstly, the Z value range of the 2nd and subsequent mixed pixels as well as the background points is estimated. Among them, the Z value range of mixed pixels is obtained from the x value range of mixed pixels and the x – Z_m relation. The Z value range of background points is estimated by selecting some background points from the scan data and finding their Z value range. Then, the scan points, whose Z values are within this range, are filtered out. It is assumed that the 1st mixed pixel in each row and valid points are not removed in this step.

**Step 2:** Remove the last scan point in each row because there is the only one mixed pixel in each row after step 1 is implemented.

In the following example, assuming that D_1 = 1.0 m, D_2 = 2.5 m, d = 4 mm and s = 1.5 mm, the x – Z_m relation is theoretically obtained and shown in Fig. 11. Because d/s falls between 2 and 3, there are 3 non-valid points in each row. According to the x – Z_m relation, the Z values of the 1st, 2nd, and 3rd mixed pixels are within [0.67 mm], [67 mm, 1480 mm], and [1480 mm, 1500 mm], respectively. In addition, it is assumed that the valid points and background points have Z values within [-10 mm, 10 mm] and [1490 mm, 1510 mm], respectively. Therefore, the developed filter first removes the scan points whose Z values are within [67 mm, 1510 mm], and then removes the last scan point in each row so that all the mixed pixels and background points are filtered out from the scan data.

The developed filter for case 2 can avoid misclassification but its performance relies on the assumption that the 1st mixed pixel in each row and the valid points are not removed in step 1. This assumption requires that the Z value range of the 1st mixed pixel and the valid points does not overlap the Z value range of the 2nd and subsequent mixed pixels and the background points. The assumption can be invalid under the following two circumstances.

1. The Z value range of the 1st mixed pixel overlaps the Z value range of the background points, making the 1st mixed pixel removed in step 1. This problem occurs when s is so large that the 1st mixed pixel has a very small x value, resulting in a mixed pixel close to the background surface. In this case, after applying the developed filter for case 2, the last point in each row can be a mixed pixel or a valid point. Hence, this mixed pixel problem is transformed from case 2 to case 1, and the filter for case 1 should then be used.

2. The Z value range of the valid points overlaps the Z value range of the 2nd and subsequent mixed pixels, making the valid points removed in step 1. This problem occurs when s is so small that the 2nd mixed pixel has a very large x value, resulting in a mixed pixel close to the foreground surface. In this case, the filtering algorithm is adjusted as follows: step 1 filters out all the non-valid points except the first M mixed pixels in each row. Now, because there are exactly M mixed pixels remained in each row after implementing step 1, step 2 further removes the last M scan points in each row. Here, M can be 2 or even larger to ensure that the valid points are not removed in step 1.

4. Experimental validation

Scanning experiments are conducted to verify the formulation of the distance measurement of mixed pixels in Section 4.1, and to examine the performance of the proposed mixed pixel filter in Section 4.2.

4.1. Validation of the distance measurement of mixed pixels

The formulation of the distance measurement of mixed pixels is verified by comparing the experimental results and the theoretical predictions of the x – D_m relation.

4.1.1. Experimental results of the x – D_m relation

A board covered with white paper was set up in front of a white wall, as shown in Fig. 12, so that mixed pixels can occur along the edges of the white board. The white board surface is defined as the foreground surface, and the white wall as the background surface. The scan data were acquired by a FARO Focus 3D 120 laser scanner (FARO, 2015), which uses three modulated waves with wave-
lengths of 307 m, 19.2 m and 2.4 m. Because the \( x - D_m \) relation changes as \( \Delta D \) changes, as described in Section 2.2.3, four scanning experiments (Exps. 1–4) were conducted with different \( \Delta D \) values. The scanning parameters of the four experiments are summarized in Table 1.

For each experiment, a row of scan points was randomly selected, and the mixed pixels in each row were manually identified for reference. For each mixed pixel, its distance measurement \( D_m \) was obtained from the scan data, and its \( x \) value was estimated based on the spacing between adjacent points and its sequence. Therefore, a set of \( x - D_m \) data was obtained from each experiment.

4.1.2. Theoretical predictions of the \( x - D_m \) relation

According to Eq. (14), \( D_m \) is a function of \( \lambda_i, D_1, D_2, \) and \( I_1/I_2 \). For each experiment, \( \lambda_i, D_1, \) and \( D_2 \) are all known, and the only unknown is \( I_1/I_2 \). For the split laser beam shown in Fig. 3(b), the emitted intensities of the laser beam falling on the foreground (\( A_1 \)) and the background (\( A_2 \)) surfaces are denoted as \( I_{e1} \) and \( I_{e2} \), respectively. The total emitted intensity is denoted as \( I_e = I_{e1} + I_{e2} \), and \( I_e \) is constant for the identical laser source. For each of the foreground and background surfaces, the ratio of the returned intensity to the emitted intensity is denoted as \( k_1 \) and \( k_2 \), respectively. Therefore, the emitted and returned intensities of the mixed pixel are related as follows:

\[
k_i = I_{r1}/I_{e1}, i = 1, 2
\]

Hence, \( I_1/I_2 \) is influenced by \( k_1/k_2 \) and \( I_{e1}/I_{e2} \). Next, these two factors are estimated to obtain \( I_1/I_2 \).

\[
\frac{I_1}{I_2} = \frac{k_1 I_{e1}}{k_2 I_{e2}}
\]

- Estimation of \( k_1/k_2 \)

The scanner provides a reflection value \( R \) for each scan point, which is related to the intensity \( I_r \) of the returned signal.

\[
\log(I_r) = V(R + Q)
\]

where \( V \) and \( Q \) are constants determined by the scanner (Kaasalainen et al., 2008).

For a valid point, the intensity of the returned signal is \( k_1 I_r \). The reflection value of the valid point is denoted as \( R_1 \). Similarly, for a background point, the returned intensity and the reflection value are \( k_2 I_r \) and \( R_2 \), respectively. Thus, according to Eq. (21):

\[
\log(k_1 I_r) = V(R_1 + Q)
\]

\[
\log(k_2 I_r) = V(R_2 + Q)
\]

Performing subtraction yields:

\[
\log\left(\frac{k_1}{k_2}\right) = V(R_1 - R_2)
\]

Because \( R_1 \) and \( R_2 \) can be obtained from the scan data by finding a valid point and a background point, respectively, \( V \) becomes the only unknown to estimate \( k_1/k_2 \), and \( V \) is estimated as follows.

1. Conduct scanning experiments on the same object (a concrete element in this study) with the same incident angle of 0° but with varying scanning distances (20–40 m). From each scan, select only one specific scan point and obtain its distance measurement \( D_i \) and reflection value \( R_i \).

2. Obtain multiple sample data sets of \( \left( \log\left(\frac{k_1}{k_2}\right), (R_1 - R_2) \right) \).

Here, two arbitrarily selected scan points (one with \( D_i \) and \( R_i \), and the other with \( D_q \) and \( R_q \)) produce one sample data of \( \left( \log\left(\frac{k_1}{k_2}\right), (R_1 - R_2) \right) \). The \( k_i \) value of each scan point is proportional to the inverse square of distance \( D_i \), i.e., \( k_i = 1/D_i^2 \) (Nitzan et al., 1977). As all the other factors influencing \( k_i \), including the diffuse reflectance of the reflecting surface and the incident angle of the laser beam, are identical for all the selected points, \( \log(\frac{k_1}{k_2}) \) can be obtained from \( \log(\frac{R_1}{R_2}) \). Note that the relation of \( k_i = 1/D_i^2 \) is valid only when the scanning distance is longer than 10 m (Kaasalainen et al., 2011). Therefore, it cannot be used to directly estimate \( k_1/k_2 \) for a short range measurement like in this study.

3. Estimate the \( V \) value by fitting the model in Eq. (24) to the multiple sample data sets of \( \left( \log\left(\frac{k_1}{k_2}\right), (R_1 - R_2) \right) \) using least squares analysis. Once the model is fitted, the \( k_1/k_2 \) value for any mixed pixel can be estimated from the given \( V, R_1, \) and \( R_2 \) values.

- Estimation of \( I_{e1}/I_{e2} \)

The value of \( I_{e1}/I_{e2} \) is related to \( x \) and how the laser power is distributed over the cross section of the laser beam. Define the laser power distribution function \( g(x) \) as the ratio \( I_{e1}/I_{e2} \).

\[
g(x) = I_{e1}/I_{e2}
\]

Because it is difficult to analytically derive \( g(x) \), an experiment is performed to estimate \( g(x) \) as follows.

1. Scan an edge that is separated by two different colors. As shown in Fig. 13, a white background is covered by a black rectangular shape.

2. Randomly select a row of scan points and find the scan points across the edge. Because the scan points falling on the white and black areas have different reflection values of 1900 and 1400, respectively, the scan points across the edge can be easily identified based on the reflection value.
For each point across the edge, estimate its \( x \) value based on the spacing between two adjacent scan points and the sequence of this point.

For each point across the edge, estimate its \( g(x) \) value. Because the returned signals from the two different colors have the same phase, the total returned intensity \( I_t \) equals to the sum of the two returned intensities:

\[
l_t = k_w I_{t1} + k_b I_{t2} = k_w d g(x) + k_b I_e (1 - g(x))
\]

where \( k_w \) and \( k_b \) are the ratios of the returned intensity to the emitted intensity for the white and black surfaces, respectively. Here, \( k_w d \) and \( k_b I_e \) represent the returned intensities when the whole laser beam falls on the white and the black surfaces, respectively. According to Eq. (21), \( k_w I_e \), \( k_b I_e \), and \( I_t \) can be obtained from the reflection values of the corresponding scan points. Therefore, the \( g(x) \) value for each \( x \) value can be estimated from Eq. (26).

(5) Based on the \( x - g(x) \) data set estimated in the previous two steps, \( g(x) \) can be obtained by fitting a polynomial equation to the data set. Then, \( l_{t1}/l_{t2} \) is obtained as \( g(x)(1 - g(x)) \).

Finally, the theoretical prediction of the \( x - D_m \) relation is derived from the estimates of \( k_1/k_2 \) and \( l_{t1}/l_{t2} \).

4.1.3. Comparison of theoretical predictions and experimental results

Fig. 14(a)–(d) compares the theoretical predictions (red curves) and the experimental results (blue dots) of the \( x - D_m \) relation for all the four experiments. The average RMS of discrepancies between the predictions and the experimental results were 5.1 mm, 6.2 mm, 2.4 mm, and 4.7 mm for Exps. (1)–(4), respectively. The magnitude of the discrepancy was at least one or two orders of magnitude smaller than the magnitudes of \( D_1 \), \( D_2 \), and \( \Delta D \). Therefore, the formulation of the distance measurement of mixed pixels, which is illustrated in Section 2, can accurately predict the locations of mixed pixels in actual scan data.

4.2. Validation of the proposed mixed pixel filter

To evaluate the performance of the proposed mixed pixel filter, scanning experiments were conducted on a test specimen and the dimensions of the specimen were estimated after applying the proposed filter. As a benchmark, the scanner's built-in filter was applied to the same scan data before estimating dimensions.

4.2.1. Test specimen and experimental set-up

The test specimen was manufactured to be rectangular with dimensions of 840 mm \( \times \) 300 mm. As shown in Fig. 15(a) and (b), two scanning experiments (Exp. 5 and Exp. 6) were conducted by taking the specimen as the foreground surface and a steel gate as the background surface. The two experiments had the same \( D_1 \) and \( D_2 \) values of 8 m and 9.5 m, but different angular resolutions of 0.045° (Exp. 5) and 0.018° (Exp. 6).

4.2.2. Applying the proposed mixed pixel filter to the scan data

Because mixed pixels occur along all four edges of the specimen, the proposed mixed pixel filter was applied to all four edges. Since the size of the laser beam is different in the horizontal and vertical directions, the filter was separately implemented in two directions.

Exp. 5: For the mixed pixels along the left and right edges of the specimen, the horizontal size of the laser beam is of concern.

Because the horizontal size of the laser beam was 5.1 mm and the spacing between adjacent points was 6.3 mm, the proposed filter for case 1 was applied to the left and right edges. The proposed filter first removed obvious non-valid points by the threshold \( T_1 \) of 500 mm. Then, the remaining scan data were classified into mixed pixels and valid points by the threshold \( T_2 \) of 2.6 mm to minimize the classification error. On the other hand, for the mixed pixels along the top and bottom edges, because the vertical size of the laser beam was 7.6 mm and the spacing between adjacent points was 6.3 mm, the proposed filter for case 2 was applied. There were 1 or 2 mixed pixels in each column of scan points. According to the theoretical \( x - Z_m \) relation, the first mixed pixel had \( Z_m \) values within \([0.1487, \infty)\) and the second within \([1487, 1500)\). Besides, sampling points from scan data showed that the valid points had \( Z_m \) values within \([-4.4, 4.4)\) and the background points had \( Z_m \) values within \([1495, 1505)\). Therefore, the filter first removed the scan points with \( Z_m \) values within \([1487, \infty)\) and then removed the last point in each column. Fig. 16 shows the scan data before and after applying the proposed filter, where the background points and mixed pixels were successfully removed.

Exp. 6: Because Exp. 6 has a higher angular resolution, the spacing between scan points was 2.5 mm, less than that in Exp. 5. Since the horizontal size of the laser beam was 5.1 mm, there were 2 or 3 mixed pixels in each row of scan points for the left and right edges, and the mixed pixel filter for case 2 was applied. Besides, because the vertical size of the laser beam was 7.6 mm, there were 3 or 4 mixed pixels in each column of scan points for the top and bottom edges, and the mixed pixel filter for case 2 was applied. Note that, in both directions, the proposed filter for case 2 was successfully implemented without encountering the two special circumstances described at the end of Section 3.2. Fig. 17 shows the scan data before and after applying the proposed filter, indicating that the background points and mixed pixels were successfully removed by the proposed filter.

4.2.3. Applying the scanner's built-in filter to the scan data

As a benchmark, the scanner's built-in mixed pixel filter was applied to the same scan data from the two experiments. The working principle of the scanner's filter is explained as follows. For each scan point being checked, the filter defines a surrounding area. For each point inside the surrounding area, its distance measurement is compared with that of the checked point, and the point is counted if the distance difference is smaller than a threshold value. If the total number of counted points is larger than another threshold value, the checked point remains in the scan data; otherwise, it is filtered out as a mixed pixel.
Fig. 14. Comparison of the theoretical predictions and experimental results of the $x - D_m$ relation with different $\Delta D$ values: (a) $\Delta D = 0.35$ m, (b) $\Delta D = 0.85$ m, (c) $\Delta D = 1.35$ m, and (d) $\Delta D = 2.15$ m.

Fig. 15. Configuration of AMCW laser scanning experiments (Exps. 5 and 6) to validate the proposed mixed pixel filter: (a) Setup of the laser scanner and the specimen and (b) schematic scanning setting.

Fig. 16. Application of the proposed mixed pixel filter to scan data of Exp. 5 ($s < d$ for top and bottom edges, and $s \geq d$ for left and right edges): (a) Before applying the filter and (b) after applying the filter.
4.2.4 Comparison of the proposed and the scanner’s built-in filters

After applying the proposed filter or the scanner’s built-in filter, the dimensions of the specimen, i.e., lengths of the left, right, top, and bottom edges, were estimated as follows. (1) For each edge, extract the edge points by finding the last point in each row (or column). For example, as shown in Fig. 18, the filled dots are the edge points of the right edge. (2) Find the least-squares fitting line of all the edge points for each edge, e.g., $l_1$ for the right edge. (3) Compensate for the edge loss. Because the edge points are valid points that are fully inside the boundary, the true right edge should be on the right of $l_1$. Thus, the edge loss compensation model developed by Tang et al. (2009) was applied, and the final estimation of the edge line was adjusted to $l_2$. (4) Once all four edge lines were estimated, the intersection points of the edge lines were extracted as corner points and the dimensions of the specimen were estimated from the locations of the corner points.

Table 2 shows the true dimensions and the estimated dimensions after applying two different filters. The true dimensions were manually measured by a ruler with the smallest division of 1 mm. The estimated dimensions were compared to the true dimensions, and the absolute values of errors are shown in the parenthesis after the estimated dimensions. For Exp. 5, the average error of the estimated four dimensions (left, right, top, and bottom dimensions) after applying the proposed filter is 1.9 mm, whereas the average error after applying the scanner’s built-in filter is 4.8 mm. For Exp. 6, the two average errors are 1.0 mm and 3.5 mm, respectively. The scanner’s built-in filter cannot remove all the mixed pixels, resulting in overestimated dimensions. In the contrast, the proposed filter can more effectively filter out mixed pixels and yield more accurate dimension estimations. The proposed filter reduced the errors in estimated dimensions by 60% and 71% for the two experiments, respectively.

For Exp. 5, after applying the proposed filter, the errors in the top and bottom dimensions (an average of 2.7 mm) are larger than the errors in the left and right dimensions (an average of 1.2 mm). Case 1 filter with certain classification errors is applied to the left and ridge edges, while case 2 filter is applied to the top and bottom edges. Therefore, it is speculated that the proposed filter may have more classification errors along the left and right edges, resulting in larger errors in the dimension estimations of the top and bottom edges.

5. Conclusions and future work

This study presents a mixed pixel filter for scan data obtained from an AMCW laser scanner to improve the accuracy of dimension estimation. This study firstly formulates the distance measurement of mixed pixels based on the working principle of the laser scanner. Then, a mixed pixel filter is developed for two different cases, when there is at most one mixed pixel in each row (case 1) and when there is at least one mixed pixel in each row (case 2).
For case 1, the proposed filtering algorithm provides the optimal threshold value that minimizes the classification errors between the valid points and mixed pixels. For case 2, the proposed filtering algorithm produces no classification error.

To verify the formulation of the distance measurement of mixed pixels, four experiments were conducted with different scanning parameters. The distance measurement of mixed pixels, represented by the $x - D_m$ relation, was obtained both theoretically and experimentally. The comparisons of the theoretical predictions and the experimental results show that the formulation derived in Section 2 can accurately predict the locations of mixed pixels in actual scan data.

To validate the proposed mixed pixel filter, two scanning experiments were conducted with different angular resolutions on the same rectangular test specimen. The proposed filter was first applied to the scan data of the specimen, and then the dimensions of the specimen were estimated. In Exp. 5, the filtering algorithm for case 1 was applied to the left and right edges, and the filtering algorithm for case 2 was applied to the top and bottom edges. In Exp. 6, the filtering algorithm for case 2 was applied to all the four edges. The scanner’s built-in filter was applied to the same scan data as a benchmark. In the two experiments, the average errors of the estimated dimensions after applying the proposed filter are 1.9 mm and 1.0 mm, respectively, which are much smaller than the errors (4.8 mm and 3.5 mm) when using the scanner’s built-in filter. It is demonstrated that the proposed filter can more effectively remove mixed pixels and provide more accurate dimension estimates.

Note that the proposed mixed pixel filter is developed with a few assumptions on $D_1$, $D_2$ and the scanner’s parameters, as described at the beginning of Section 2.2.3. Therefore, its application is limited within the boundaries of these assumptions. A similar principle may be applied to develop another mixed pixel filter for different circumstances.

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